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Table of Contents

Types, Abstraction, and Parametric Polymorphism, Part 2 1
QingMing Ma and John C. Reynolds

Call-by-Value Combinatory Logic and the Lambda-Value Calculus 41
John Gateley and Bruce F. Duba

From Operational to Denotational Semantics 54
Scott F. Smith

A Simple Language Supporting Angelic Nondeterminism and Parallel Composition 77
Michael W. Mislove and Frank J. Oles

The Equivalence of Two Semantic Definitions for Inheritance ....
in Object-Oriented Languages 102
Kim B. Bruce

Primitive Recursive Functionals with Dependent Types 125
Neal Nelson

Typed Homomorphic Relations Extended with Subtypes 144
Gary T. Leavens and Don Pigozzi

Information Links in Domain Theory 168
Jon Barwise

Nonwellfounded Sets and Programming Language Semantics 193
J. J. M. M. Rutten

Simultaneous Substitution in the Typed Lambda Calculus 207
John W. Gray

HSP Type Theorems in the Category of Posets 221
Michael Barr

Decomposition of Domains 235
Achim Jung, Leonid Libkin and Hermann Puhlmann

Cartesian Closed Categories of Domains and the Space Proj(D) 259
Michael Huth

An Upper Power Domain Construction in Terms of Strongly Compact Sets 272
Reinhold Heckmann

Correctness of Procedure Representations in Higher-Order Assembly Language 294
Mitchell Wand

An Algorithm for Analyzing Communicating Processes 312
Nicolas Mercouroff

Continuous Functions and Parallel Algorithms on Concrete Data Structures 326
Stephen Brookes and Shai Geva

Trade-Offs in True Concurrency: Pomsets and Mazurkiewicz Traces 350
Bard Bloom and Marta Kwiatkowska

On Relating Concurrency and Nondeterminism 376
Luca Aceto

On Continuous Time Agents 403
Stefano Kasangian

A Monoidal Closed Category of Event Structures 426
Guo-Qiang Zhang

An Exper Model for Quest 436
Giuseppe Rosolini

Equations for if-then-else 446
Ernest G. Manes

Program Correctness and Matricial Iteration Theories 457
Stephen L. Bloom and Zoltán Ésik

Liminf Progress Measures 477
Nils Klarlund

Connections 492
Austin Melton, Bernd S. W. Schroeder and George E. Strecker

Types, Abstraction, and Parametric Polymorphism, Part 2*

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Abstract

The concept of relations over sets is generalized to relations over an arbitrary category, and used to investigate the abstraction (or logical-relations) theorem, the identity extension lemma, and parametric polymorphism, for Cartesian-closed-category models of the simply typed lambda calculus and PL-category models of the polymorphic typed lambda calculus. Treatments of Kripke relations and of complete relations on domains are included.

In [1], the idea that type structure enforces abstraction was formalized by an "abstraction theorem" that was proved for both the simply typed (or first-order) lambda calculus and the polymorphic (or second-order) lambda calculus [2, 3, 4]. In the polymorphic case this theorem led naturally to a definition of "parametric" polymorphism that captured the intuitive concept first described by Strachey [5]. Unfortunately, however, most of the results of [1] were limited to a classical set-theoretic model. Thus the abstraction theorem for the simply typed case was merely a repetition of the logical-relations theorem for the typed lambda-calculus [6], while the developments for the polymorphic case were vacuous, since it was later shown that there is no classical set-theoretic model of the polymorphic lambda calculus [7, 8].

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[7] P.-L. Curien. *Categorical Combinators, Sequential Algorithms and Functional Programming*. Pitman, 1986.

[8] G. Gierz, K. H. Hofmann, K. Keimel, J. D. Lawson, M. Mislove, and D. S. Scott. *A Compendium of Continuous Lattices*. Springer Verlag, Berlin, 1980.

[9] J.-Y. Girard. The System F of Variable Types: Fifteen Years Later. *Theoretical Computer Science*, 45:159–192, 1986.

[10] G. Grätzer. *General Lattice Theory*. Birkhäuser Verlag, Basel, 1978.

[11] G. Grätzer. The complete congruence lattice of a complete lattice. *Proc. of Int. Conf. on Lattices, Semigroups, and Universal Algebra*. Plenum Press, New York (1990), 81–88.

[12] G. Grätzer and E. T. Schmidt. A representation of m–algebraic lattices. *Algebra Universalis*, to appear.

[13] A. Jung. *Cartesian Closed Categories of Domains*, volume 66 of *CWI Tracts*. Centrum voor Wiskunde en Informatica, Amsterdam, 1989.

[14] G. Kahn and G. Plotkin. Domaines Concrets. Technical Report 336, INRIA-Laboria, 1978.

[15] A. Kanda. Fully Effective Solutions of Recursive Domain Equations. In J. Beçvar, editor, *Mathematical Foundations of Computer Science*, volume 74. Springer-Verlag, 1979. Lecture Notes in Computer Science.

[16] K. G. Larsen and G. Winskel. Using Information Systems to Solve Recursive Domain Equations Effectively. In G. Kahn, D. B. MacQueen, and G. Plotkin, editors, *Semantics of Data Types*, volume 173 of *Lecture Notes in Computer Science*, pages 109–130. Springer-Verlag, 1984.

[17] L. Libkin. A relational algebra for complex objects based on partial information. In: J. Demetrovics and B. Thalheim, editors, *Mathematical Fundamentals of Database Systems – 91*, volume 495 of *Lecture Notes in Computer Science*, pages 36–41. Springer-Verlag, 1991.

[18] G. D. Plotkin. Post-Graduate Lecture Notes in Advanced Domain Theory (incorporating the "Pisa Notes"). Dept. of Computer Science, Univ. of Edinburgh, 1981.

[19] H. Puhlmann. Verallgemeinerung relationaler Schemata in Datenbanken mit Informationsordnung, 1990. (Diplomarbeit, Technische Hochschule Darmstadt.).

[20] M. B. Smyth. Effectively Given Domains. *Theoretical Computer Science*, 5:257–274, 1977.

[21] P. Taylor. Homomorphisms, Bilimits and Saturated Domains. Some Very Basic Domain Theory. Draft, Imperial College London, 15pp., 1987.

[22] K. Weihrauch and T. Deil. Berechenbarkeit auf cpo's. Technical Report 63, Rheinisch-Westfälische Technische Hochschule Aachen, 1980.

[23] G. Winskel. *Events in Computation*. PhD thesis, University of Edinburgh, 1981.

Cartesian Closed Categories of Domains and the Space Proj(D)

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Abstract

This paper studies algebraic dcpos and their space $[D \xrightarrow{\text{pr}} D]$ of *Scott-continuous projections*. Let $\text{ALG}_\perp$ be the category of algebraic dcpos with bottom and Scott-continuous maps as arrows. If $\mathcal{C}$ is a full cartesian closed subcategory of $\text{ALG}_\perp$ such that $\mathcal{C}$ is closed under $D \mapsto [D \xrightarrow{\text{pr}} D]$, then every object D is *projection-stable*, i.e., $\text{im}(p)$ is algebraic for all $p \in [D \xrightarrow{\text{pr}} D]$. This is equivalent to assuming that all order-dense chains in $\text{K}(D)$ are degenerate. If $\mathcal{C}$ contains an isomorphic copy of the flat natural numbers, then $\mathcal{C}$ is *not* closed under finitary Scott-continuous retractions. In this case, $\mathcal{C}$ contains an object D, such that $\text{K}(D)$ is not a lower set in D. This puts serious constraints on the existence of cartesian closed categories closed under $D \mapsto [D \xrightarrow{\text{pr}} D]$. The cartesian closed category of dI-domains, however, is closed under $D \mapsto \text{Retr}(D)$ and $D \mapsto \text{Proj}(D)$, where $\text{Retr}(D)$ is the dI-domain of all *stable* Scott-continuous retractions in the *stable* order. The dI-domain $\text{Proj}(D)$ consists of all $p \in \text{Retr}(D)$ which are below the identity in the stable order. All dI-domains are projection-stable, and $\text{Proj}(D)$ is isomorphic to the Hoare power domain of the ideal completion of the poset of complete primes of D. It is therefore a completely distributive bialgebraic lattice and all maps $f\colon D \to E$ into complete lattices E preserving all existing suprema have unique extensions to $\text{Proj}(D)$, where $\lambda c.\lambda x.x \sqcap c$ is the embedding of D into $\text{Proj}(D)$.

0 Notation and Basic Definitions

The reader should scan this paragraph to check whether the notational conventions deviate from his or her standards. A *directed* set A in a poset P is a non-empty set such that for all $a, b \in A$ there exists some $c \in A$ such that $a, b \leq c$. If $X \subseteq P$ is given, then an *upper bound* for X in P in an element $u \in P$ such that $x \leq u$ for all $x \in X$. If there exists an upper bound s of X such that $s \leq u$ for all upper bounds u of X, then s is called the *supremum* of X. We write $s = \sqcup X$. A *dcpo* D is a poset in which every directed subset A has a supremum. We say that x is *way-below* y in D iff for all directed subsets $A \subseteq D$, the relation $y \leq \sqcup A$ implies $x \leq a$ for some $a \in A$. We denote this by $x \ll y$.

A dcpo is *continuous* iff for all $x \in D$, the set of all y such that $y \ll x$ is directed and its supremum equals x. The elements $x \ll x$ in D are called *finite*, and $\mathrm{K}(D)$ denotes the poset of finite elements of D.

A dcpo D is *algebraic* iff for all $x \in D$, the set of all $k \in \mathrm{K}(D)$ with $k \le x$ is directed, and the supremum of this set equals x. Note that every algebraic dcpo is continuous. A map $f: D \to E$ between dcpos D and E is *Scott-continuous* iff $\bigsqcup f(A) = f(\bigsqcup A)$ for all directed sets $A \subseteq D$. We write id_D for the identity function on D. Let $D \to E$ denote the dcpo of all Scott-continuous maps $f: D \to E$ in the pointwise order.

If P is a poset and $x \in X \subseteq P$, we write $\downarrow(x)$ for the set of all $y \in P$ with $y \le x$. Then, $\downarrow(X) = \bigcup\{\downarrow(x) \mid x \in X\}$ is well-defined. The meanings of $\uparrow(x)$ and $\uparrow(X)$ should now be obvious. A *minimal upper bound* of X is an upper bound m of X such that $u \le m$ implies $u = m$ for all upper bounds u of X. A poset P is a *complete lattice* iff every subset $X \subseteq P$ has a supremum in P. A subset $C \subseteq P$ is a *chain* iff for all $a, b \in C$, we have $a \le b$ or $b \le a$. A set X is *bounded* in P iff it has an upper bound in P. A poset P is *locally finite* iff $\downarrow(p)$ is finite for all $p \in P$. A subset $N \subseteq P$ is *normal* in P iff $\downarrow(x) \cap N$ is directed for all $x \in P$. A map $f: P \to Q$ between two posets P and Q *preserves all existing suprema* iff for all $Y \subseteq P$ such that $\bigsqcup Y$ exists, we have $\bigsqcup f(Y)$ exists and $f(\bigsqcup Y) = \bigsqcup f(Y)$.

An element $\bot \in D$ with $\bot \le x$ for all $x \in D$ is called a *bottom* of D. A *Scott-domain* D is an algebraic dcpo with bottom such that every bounded subset of D has a supremum in D. A dcpo D is *bialgebraic* iff D and D^{op} (D in the opposite order) are algebraic. A complete lattice D is *completely distributive* iff for all index sets I and J and for all families $(x_{ij})_{(i,j) \in I \times J}$ in D we have

$$\sqcap_{i\in I} \bigsqcup_{j \in J} x_{ij} = \bigsqcup_{f \in J^I} \sqcap_{i \in I} x_{if(i)},$$

where J^I is the set of all functions $f: I \to J$.

1 Introduction

This paper deals with *algebraic dcpos* and focuses on a special class of morphisms, *Scott-continuous projections* $p: D \to D$. Those are maps which are idempotent and below the identity

$$(1) \qquad p \circ p = p \le \mathrm{id}_D.$$

Let $[D \xrightarrow{\mathrm{pr}} D]$ denote the dcpo of all such Scott-continuous maps $p: D \to D$ in the pointwise order. It is a standard result that the image of p, $\mathrm{im}(p)$, is a *continuous dcpo*, if D is one [Jun 1989]. However, if D is algebraic and $p \in [D \xrightarrow{\mathrm{pr}} D]$ is given, then $\mathrm{im}(p)$ is not necessarily algebraic. This has been known implicitely for quite a while [Can 1884]. Let us denote by $\mathrm{F}_{\mathrm{pr}}(D)$ the space of all those $p \in [D \xrightarrow{\mathrm{pr}} D]$ where $\mathrm{im}(p)$ is algebraic; we call $p \in \mathrm{F}_{\mathrm{pr}}(D)$ a *finitary* projection. The stucture of $\mathrm{F}_{\mathrm{pr}}(D)$ has been studied in [Gun 1985] and $\mathrm{F}_{\mathrm{pr}}(D)$ has been used for various purposes in denotational semantics [Gun et.al. 1990].

A first immediate question is, when do we have

$$(2) \qquad \mathrm{F}_{\mathrm{pr}}(D) = [D \xrightarrow{\mathrm{pr}} D].$$

If D fulfilles (2), we call D *projection-stable*. A second question is whether $[D \xrightarrow{\mathrm{pr}} D]$ is an algebraic dcpo—or are there even cartesian closed categories of algebraic dcpos which are closed under $D \mapsto [D \xrightarrow{\mathrm{pr}} D]$?

We will demonstrate how closely related all these questions are. We will further come up with some evidence that the last question has only trivial (i.e., finite) solutions. However, in the cartesian closed category of *dI-domains*, the space $\mathrm{Proj}(D)$ of all *stable* functions below the identity in the *stable* order is a dI-domain, and dI-domains are projection-stable. We begin with some basic facts.

1 Lemma. Let D be a dcpo, $p \in [D \xrightarrow{\mathrm{pr}} D]$, and $A \subseteq [D \xrightarrow{\mathrm{pr}} D]$ directed. Then we have the following:

(i) For $x, y \in \mathrm{im}(p)$, x is way-below y in D iff x is way-below y in $\mathrm{im}(p)$.

(ii) $\mathrm{K}(\mathrm{im}(p)) = \mathrm{K}(D) \cap \mathrm{im}(p)$.

(iii) For $\bigsqcup A: D \to D$, $(\bigsqcup A)(x) := \bigsqcup\{p(x) \mid p \in A\}$, we have that $\bigsqcup A$ is the supremum of A in $[D \xrightarrow{\mathrm{pr}} D]$.

(iv) $\mathrm{K}(\mathrm{im}(\bigsqcup A)) = \bigcup_{p \in A} \mathrm{K}(\mathrm{im}(p))$.

PROOF. If $x, y \in \mathrm{im}(p)$ are given, then $x \ll y$ in D implies $x \ll y$ in $\mathrm{im}(p)$, as $\mathrm{im}(p)$ is closed under directed suprema in D. Conversely, for $x \ll y$ in $\mathrm{im}(p)$ and $B \subseteq D$ directed such that $y \le \bigsqcup B$, the set $p(B) \subseteq \mathrm{im}(p)$ is directed and $y \le \bigsqcup p(B)$, as p is Scott-continuous. Hence, $x \le p(b) \le b$ follows for some $b \in B$. Now, (ii) is a direct consequence of (i), and for (iii) it suffices to show $\bigsqcup A \in [D \xrightarrow{\mathrm{pr}} D]$, which is immediate. For (iv), let $p \in A$ and $k \in \mathrm{K}(\mathrm{im}(p)) = \mathrm{K}(D) \cap \mathrm{im}(p)$. Then, k is a fixed point of $\bigsqcup A$ and finite. By (ii), k is in $\mathrm{K}(\mathrm{im}(\bigsqcup A))$. Conversely, if $l \in \mathrm{K}(\mathrm{im}(\bigsqcup A))$ is given, then $l = \bigsqcup\{q(l) \mid q \in A\}$ and $l \in \mathrm{K}(D)$ imply that $l \le q(l) \le l$ for some $q \in A$. Hence, $l \in \mathrm{K}(D) \cap \mathrm{im}(q) = \mathrm{K}(\mathrm{im}(q))$. □

Next we need to know how $\mathrm{F}_{\mathrm{pr}}(D)$ sits inside of $[D \xrightarrow{\mathrm{pr}} D]$. Recall that a continuous dcpo E is algebraic iff [Hut 1990]

$$(3) \qquad (\forall x, y \in E): \ x \ll y \text{ in } E \Rightarrow (\exists k \in \mathrm{K}(D): \ x \le k \le y).$$

A continuous dcpo E also satisfies the *interpolation property* [Jun 1989]

$$(4) \qquad (\forall x, y \in D): \ x \ll y \text{ in } E \Rightarrow (\exists z \in D: \ x \ll z \ll y).$$

2 Lemma. Let D be an algebraic dcpo. Then the inclusion map $F_{pr}(D) \to [D \xrightarrow{pr} D]$ is Scott-continuous.

PROOF. Let $A \subseteq F_{pr}(D)$ be directed. We have to show that $\sqcup A \in F_{pr}(D)$. Let $x \ll y$ be given in the continuous dcpo $im(\sqcup A)$ and interpolate such that $x \ll z \ll y$ in $im(\sqcup A)$. Then, $z \ll y = \sqcup\{p(y) \mid p \in A\}$ implies $z \leq p(y)$ for some $p \in A$. As $x \ll z$ in D, we have $x \ll p(y)$ in D. Thus, $x \leq k \leq p(y)$ for some $k \in K(im(p)) \subseteq K(im(\sqcup A))$, as $im(p)$ is algebraic. Hence, $x \leq k \leq y$ and (3) do it. □

2 Algebraic dcpos and the Space $[D \xrightarrow{pr} D]$

Recall that a poset P has ***property m*** [Gun 1985] iff for every finite set $F \subseteq P$, all upper bounds of F are above some minimal upper bound of F.

An ***algebraic L-domain*** [Jun 1989] is an algebraic dcpo D such that $\downarrow(x)$ is a complete lattice for all $x \in D$. A ***profinite dcpo*** E [Gun 1985] is an algebraic dcpo such that

$$(5) \qquad id_E = \bigsqcup\nolimits^{\uparrow}_{i \in I} d_i,$$

where $(d_i)_{i \in I}$ is a directed family in $F_{pr}(D)$ and $im(d_i)$ is finite for all $i \in I$.

3 Lemma. If D is an algebraic L-domain or profinite, then $K(D)$ has property m. In this case, $F_{pr}(D)$ is an algebraic lattice.

PROOF. Gunter has shown that $F_{pr}(D)$ is an algebraic lattice, if $K(D)$ has property m (using an isomorphic copy of $F_{pr}(D)$ [Gun 1985]). If D is a profinite dcpo, then $K(D)$ is known to have property m [Gun 1985]. For an L-domain D, the poset $K(D)$ has property m, as all $\downarrow(k)$, $k \in K(D)$, are complete lattices. □

Now we are in a position to specify when $[D \xrightarrow{pr} D]$ is algebraic.

4 Theorem. Let $\mathcal{C}$ be a cartesian closed full subcategory of $ALG_\perp$. If D is an object of $\mathcal{C}$, then the following are equivalent:

(i) $[D \xrightarrow{pr} D]$ is algebraic.

(ii) $[D \xrightarrow{pr} D]$ is an algebraic lattice.

(iii) D is projection-stable.

PROOF. By Achim Jung's classification [Jun 1989], D is an algebraic L-domain or profinite. Hence, Lemma 3 shows that (iii) implies (ii). Clearly, (ii) implies (i). By Lemma 2, it suffices to show

$$(6) \qquad K([D \xrightarrow{pr} D]) \subseteq F_{pr}(D)$$

to complete the proof.

Let $p \in K([D \xrightarrow{pr} D])$ be given. If D is profinite, we use (5) and have $p \leq \sqcup_{i \in I} d_i$, which implies $p \leq d_j$ for some $j \in I$. Therefore, $im(p)$ is finite, so $p \in F_{pr}(D)$ follows. If D is an algebraic L-domain, let $z \in im(p)$ be given. Then, $E := \downarrow(z)$ is an algebraic lattice with $K(E) \subseteq K(D)$. If $A \subseteq E$ is directed with $z \leq \sqcup_E A$, define for all $a \in A$

$$(7) \qquad p_a(x) := \begin{cases} x, & \text{if } x \notin E, \\ x \sqcap a, & \text{otherwise,} \end{cases}$$

where $\sqcap : E \times E \to E$ is the infimum operation in the algebraic lattice E. Then, the set $\{p_a \mid a \in A\}$ is directed in $[D \xrightarrow{pr} D]$, and $p \leq id_D = \sqcup\{p_a \mid a \in A\}$ implies $p \leq p_a$ for some $a \in A$. Now, $z = p(z) \leq p_a(z) = z \sqcap a \leq a$ shows $z \in K(E)$. Therefore, $K(im(p)) = K(D) \cap im(p) = im(p)$ proves $p \in F_{pr}(D)$. □

Theorem 4 is unsatisfactory, as we do not know when D is projection-stable if we look at $K(D)$ only. This will be handled next. A chain C is *order-dense* in a poset P iff $a < b$ in C implies $a < c < b$ for some $c \in C$. An order-dense chain C is *degenerate* iff it contains at most one element.

5 Theorem. Let D be an algebraic dcpo with bottom. Then the following are equivalent:

(i) D is projection-stable.

(ii) All order-dense chains in $K(D)$ are degenerate.

PROOF. If $p \in [D \xrightarrow{pr} D] \setminus F_{pr}(D)$ is given, we construct inductively a non-degenerate chain $C \subseteq K(D)$. As $im(p)$ is not algebraic, we have $x \ll y$ in $im(p)$ such that

$$(8) \qquad x \leq m \leq y \Rightarrow m \notin K(im(p)),$$

as a consequence of (3). We interpolate with some $z_1, z_2 \in im(p)$ using (4) twice, and we obtain $x \ll z_1 \ll z_2 \ll y$. By Lemma 1, this relation holds in D, too. As D is algebraic there exists $\{k_1, k_2, k_3\} \subseteq K(D)$ such that $x \leq k_1 \leq z_1 \leq k_2 \leq z_2 \leq k_3 \leq x$. As $im(p)$ is continuous and $\{k_1, k_2, k_3\} \subseteq K(D)$, we have $z_1^*, z_2^* \in im(p)$ such that $k_1 \leq z_1^* \ll z_1 \leq k_2 \leq z_2^* \ll z_2 \leq k_3$. With the same reasoning as before we can put elements k_4, k_5 of $K(D)$ between z_i^* and z_i for $i = 1, 2$. We have a finite chain $k_1 \leq k_4 \leq k_2 \leq k_5 \leq k_3$ with k_1 as minimal and k_3 as maximal element. Further, we can always change expressions of the form

$$(9) \qquad k \leq z \leq k^*$$

into

$$(10) \qquad k \leq z^* \ll z \leq k^*$$

and fill in some $l_k \in K(D)$ such that

$$(11) \qquad k \leq z^* \leq l_k \leq z \leq k^*,$$

where $z, z^* \in im(p)$ and k^* is k's successor in the finite chain so far constructed. But the expression (11) has the structure of (9), so this process goes on ad infinitum, where we fill in 2^n finite elements at the n-th step.

What if $k = k^*$ at any finite step? Using (9), we would have $z = k \in K(D) \cap \mathrm{im}(p) = K(\mathrm{im}(p))$, which contradicts (8). Hence, the constructed chain $C \subseteq K(D)$ is non-degenerate, but C is evidently order-dense.

Conversely, let $C \subseteq K(D)$ be a non-degenerate order-dense chain. We define a map $\bar{g}: K(D) \to D$, $\bar{g}(x) := \sqcup\{c \in C \mid c < k\}$ which is well-defined and monotone. Its Scott-continuous extension $g: D \to D$ with

$$(12) \qquad g(x) := \bigsqcup\nolimits^{\uparrow}_{k \in \downarrow x \cap K(D)} \bar{g}(k)$$

is below the identity. Then, $p := g \circ g \leq \mathrm{id}_D$ is Scott-continuous. For $k \neq \bot_D$, we have

$$(13) \qquad p(k) = \bigsqcup\nolimits^{\uparrow}_{c \in C \& c < k} \bigsqcup\nolimits^{\uparrow}_{d \in C \& d < c} d, \quad \text{since } C \subseteq K(D), \text{ and therefore,}$$

$$p^2(k) = \bigsqcup\nolimits^{\uparrow}_{c \in C \& c < k} \bigsqcup\nolimits^{\uparrow}_{d \in C \& d < c} \bigsqcup\nolimits^{\uparrow}_{e \in C \& e < d} \bigsqcup\nolimits^{\uparrow}_{f \in C \& f < e} f,$$

using (12) and the continuity of p. Yet, $f < c < k$ with $f, c \in C$ implies the existence of some $d, e \in C$ with $f < e < d < c < k$, as C is order-dense.

Thus, $p(p(k)) = p(k)$ implies $p \circ p = p$, as p is Scott-continuous and D algebraic. By (13), $k = p(k)$ implies $k \leq d < c < k$ for some $c, d \in C$. Therefore, $K(\mathrm{im}(p)) = \{\bot_D\}$ and $|\mathrm{im}(p)| \geq \aleph_0$ show that p is in $[D \xrightarrow{\mathrm{pr}} D] \setminus F_{\mathrm{pr}}(D)$. □

Note that (ii) always implies (i), whereas the converse makes crucial use of the bottom element. There exists a projection-stable dcpo D, such that $K(D)$ contains non-degenerate order-dense chains [Hut 1990]. Also, if $(D_i)_{i \in I}$ is a family of projection-stable dcpos with bottom, we can use Theorem 5 to prove that $\prod D_i$ is projection-stable [Hut 1990].

Every algebraic dcpo D such that $K(D)$ is a lower set in D (or locally finite) is projection-stable (by Theorem 5). However, the property of having a locally finite or lower set $K(D)$ is not preserved by $D \mapsto (D \to D)$. The next Proposition combines this with Theorem 4 to put some constraints on cartesian closed full subcategories of $\mathrm{ALG}_\bot$ which are closed under $D \mapsto [D \xrightarrow{\mathrm{pr}} D]$.

6 Proposition. Let $\mathcal{C}$ be a full cartesian closed subcategory of $\mathrm{ALG}_\bot$ which is closed under $D \mapsto [D \xrightarrow{\mathrm{pr}} D]$. Then we have the following:

(i) Every object D of $\mathcal{C}$ is projection-stable, i.e., all order-dense chains in $K(D)$ are degenerate.

(ii) If $\mathcal{C}$ contains an isomorphic copy of the flat natural numbers, then $\mathcal{C}$ is not closed under finitary Scott-continuous retractions. Moreover, there exists an object E in $\mathcal{C}$ such that $K(E)$ is not a lower set in E. In particular, $K(E)$ is not locally finite.

PROOF. The first part is a consequence of Theorems 4 and 5. For (ii), the exponential operator in $\mathcal{C}$ is naturally isomorphic to the function space of all Scott-continuous maps [Smy 1983]. If D is an isomorphic copy of the flat natural numbers in $\mathcal{C}$, then $D_1 := [D \xrightarrow{\mathrm{pr}} D]$ is in $\mathcal{C}$ and isomorphic to the power set of the natural numbers. Thus, every countably based algebraic lattice is isomorphic to the image of a finitary Scott-continuous retraction on D_1 [Gun et.al. 1990]. But the Cantor set is such a lattice which is *not* projection-stable [Hut 1990].

Further, there exists a strictly increasing sequence $(k_n)_{n \geq 1}$ in $K(D_1)$, as D_1 is isomorphic to the powerset of the natural numbers. If D is a dcpo with bottom and $k, l \in K(D)$ are given, the function $k \searrow l$, which maps $\uparrow(k)$ to l and its complement to $\bot_D$, is a finite element of $D \to D$. Therefore, the sequence $(k_n \searrow k_2)_{n \geq 1}$ is strictly decreasing in $K(D_1 \to D_1)$ and $D_2 := D_1 \to D_1$ is an object of $\mathcal{C}$ such that $K(D_2)$ is not locally finite. Let $(l_n)_{n \geq 1}$ be a strictly decreasing sequence in $K(D_2)$. Then, $(l_n \searrow l_1)_{n \geq 1}$ is a strictly increasing sequence in $K(D_2 \to D_2)$ which is bounded by the finite function $\bot_{D_2} \searrow l_1$ in $K(D_2 \to D_2)$. Clearly, the supremum of the family $(l_n \searrow l_1)_{n \geq 1}$ cannot be a finite element, and it is therefore a non-finite element strictly below the finite element $(\bot_{D_2} \searrow l_1)$. Thus, $E := D_2 \to D_2$ is in $\mathcal{C}$ and $K(E)$ is not a lower set in E. □

3 dI-domains and the Space Proj(D)

We outlined how difficult it is to obtain cartesian closed full subcategories of $\mathrm{ALG}_\bot$ closed under $D \mapsto [D \xrightarrow{\mathrm{pr}} D]$. Why shouldn't we consider $D \mapsto F_{\mathrm{pr}}(D)$ instead (note that $F_{\mathrm{pr}}(D)$ is an algebraic lattice)?

Let us mention two objections. If p denotes a Scott-continuous projection, we would have to ensure that p is in $F_{\mathrm{pr}}(D)$, which can be a non-trivial matter (see Theorem 5). Also, the choice of $F_{\mathrm{pr}}(D)$ is not motivated by the particular category. One would expect to allow all morphisms $p^2 = p \leq \mathrm{id}$ in the order induced by the function space. We take this approach as a definition of the space $\mathrm{Proj}(D)$ in the case of dI-domains.

A *dI-domain* D is an algebraic dcpo with bottom in which every bounded set $X \subseteq D$ has a supremum in D such that the following axioms are satisfied:

(d) For all $x, y, z \in D$, we have $x \sqcap (y \sqcup z) = (x \sqcap y) \sqcup (x \sqcap z)$, whenever $y \sqcup z$ exists.

(I) $K(D)$ is locally finite, i.e., $\downarrow(k)$ is a finite set for all $k \in K(D)$.

For dI-domains D and E, the set of morphisms is the dcpo of all $f \in D \to E$ such that

$$(14) \qquad (\forall x, y, z \in D): \ x, y \leq z \Rightarrow f(x \sqcap y) = f(x) \sqcap f(y)$$

is valid. Such functions are called *stable*, and they are ordered in the *stable ordering*

$$(15) \qquad f \sqsubseteq g :\Leftrightarrow ((\forall x, y \in D): \ x \leq y \Rightarrow f(x) = f(y) \sqcap g(x)).$$

Note that $f \sqsubseteq g$ implies $f \leq g$. Moreover, we have

$$(16) \qquad (f, g \sqsubseteq h \ \& \ f \leq g) \Rightarrow f \sqsubseteq g.$$

If $\mathrm{dI}(D,E)$ is the set of stable functions in the stable order, then $\mathrm{dI}(D,E)$ is a dI-domain, and dI-domains form a cartesian closed category [Berr 1978]. For projections this naturally leads to stable projections which are below the identity in the stable order.

7 Definition. Let D be a dI-domain. Then, $\mathrm{Retr}(D)$ is the ***retraction space*** of all stable maps $r \circ r = r: D \to D$, where $\mathrm{Retr}(D)$ is endowed with the stable order. The ***projection space*** of D consists of all $r \in \mathrm{Retr}(D)$ with $r \sqsubseteq \mathrm{id}_D$ in the induced order of $\mathrm{Retr}(D)$. We denote this projection space by $\mathrm{Proj}(D)$.

We quote a result shown by Stephano Berardi [Bera 1991], because it offers us a shortcut to the central issues of the next section.

8 Theorem. [Bera 1991] Let D be a dI-domain. Then we have the following:

(i) For $r \in \mathrm{Retr}(D)$, $\mathrm{im}(r)$ is a dI-domain.

(ii) $\mathrm{Retr}(D)$ is a dI-domain. □

The next Corollary follows directly from Theorem 8, but it has been shown independently in [Hut 1990].

9 Corollary. Let D be a dI-domain. Then, $\mathrm{Proj}(D)$ is a dI-domain.

PROOF. For the dI-domain $E := \mathrm{Retr}(D)$, the map $\lambda r.r \sqcap \mathrm{id}_D$ is in $\mathrm{Retr}(E)$, where $\sqcap$ is the infimum of E. By Theorem 8, its image $\mathrm{Proj}(D)$ is a dI-domain. □

10 Corollary. The category of dI-domains is cartesian closed, it contains the flat natural numbers, and it is closed under the following operations:

(i) $D \mapsto \mathrm{Retr}(D)$.

(ii) $D \mapsto \mathrm{Proj}(D)$.

(iii) $r \mapsto \mathrm{im}(r)$, for $r \in \mathrm{Retr}(D)$. □

This result should be compared to the content of Proposition 6. Further, if $f \sqsubseteq \mathrm{id}_D$ is a stable function, it is easily seen that $\mathrm{fix}(f) := \{x \in D \mid f(x) = x\}$ is a non-empty ***Scott-closed*** subset of D (a lower set closed under directed suprema in D), and therefore $\mathrm{fix}(f)$ is a dI-domain. If D is a Scott-domain, which is not projection-stable, define $E := D \to D$ and $\Omega(g) := (g \sqcap \mathrm{id}_D)^2$. Then $\Omega \leq \mathrm{id}_E$ is Scott-continuous, yet, $\mathrm{fix}(\Omega) = [D \xrightarrow{\mathrm{pr}} D]$ is not even continuous. To see this, let p be way-below id_D in $[D \xrightarrow{\mathrm{pr}} D]$. As D is profinite, p is below some $d \in \mathrm{F}_{\mathrm{pr}}(D)$ with finite image, so $\mathrm{im}(p)$ is algebraic. If $q \in [D \xrightarrow{\mathrm{pr}} D]$ is the directed supremum of projections p' way-below q in $[D \xrightarrow{\mathrm{pr}} D]$, then we have just noted that each p' is in $\mathrm{F}_{\mathrm{pr}}(D)$, so $q \in \mathrm{F}_{\mathrm{pr}}(D)$ follows by Lemma 2. Thus, $[D \xrightarrow{\mathrm{pr}} D] = \mathrm{fix}(\Omega)$ is *not* continuous, for otherwise D would be projection-stable.

4 The Universality of Proj(D)

Let $\mathrm{Pr}(D)$ be the poset of ***complete primes*** of D:

$$(17) \qquad p \in \mathrm{Pr}(D) :\Leftrightarrow ((\forall X \subseteq D) : p \leq \sqcup X \Rightarrow (\exists x \in X : p \leq x)).$$

Every dI-domain meets the following axiom [Win 1988]:

(Pr) For all $x \in D$, $x = \bigsqcup(\downarrow(x) \cap \mathrm{Pr}(D))$.

We call a pair $(\mathcal{U}_D, i_D)$ ***universal*** for D iff the following three conditions are fulfilled:

(i) $\mathcal{U}_D$ is a dI-domain with top—and therefore an algebraic lattice.

(ii) The map $i_D : D \to \mathcal{U}_D$ preserves all existing suprema.

(iii) For all complete lattices E and all maps $f : D \to E$ preserving all existing suprema, there exists a unique map $\bar{f} : \mathcal{U}_D \to E$ preserving all suprema such that $\bar{f} \circ i_D = f$.

$$\begin{array}{ccc} D & \xrightarrow{\forall f \text{ pres. } \sqcup} & E \\ {\scriptstyle i_D}\downarrow & \nearrow {\scriptstyle \exists!\,\bar{f} \text{ pres. } \sqcup} & \\ \mathcal{U}_D & & \end{array}$$

Such an object $\mathcal{U}_D$ is unique up to isomorphism, and it can be shown to be isomorphic to the Hoare power domain of the ideal completion of the poset of complete primes of D [Hut 1991]. Moreover, $\mathcal{U}_D$ is a completely distributive algebraic lattice with $\mathrm{Pr}(\mathcal{U}_D) \cong \mathrm{Pr}(D)$ [Hut 1991]. We conclude our discussion with the following theorem.

11 Theorem. Let D be a dI-domain. Then, $(\mathrm{Proj}(D), \lambda c.\lambda x.x \sqcap c)$ is universal for D. □

First, we prove the existence of universal objects and show some of their elementary properties. We denote by $\mathcal{L}(\mathrm{Pr}(D))$ the lattice of lower sets of $\mathrm{Pr}(D)$ ordered by set inclusion. Recall that $L \subseteq \mathrm{Pr}(D)$ is a ***lower set*** in $\mathrm{Pr}(D)$ iff $x \leq y \in L$ and $x \in \mathrm{Pr}(D)$ imply $x \in L$.

12 Lemma. Let D be a dI-domain. Set $\mathcal{U} := \mathcal{L}(\mathrm{Pr}(D))$ and define a map $i: D \to \mathcal{U}$ by $i(x) := D_x$, where D_x is the set of all complete primes of D below x (we use this convenient notation throughout the paper). Then we have the following:

(i) $(\mathcal{U}, i)$ is universal for D.

(ii) $\mathrm{Pr}(\mathcal{U}) = i(\mathrm{Pr}(D)) \cong \mathrm{Pr}(D)$.

(iii) i is an order-isomorphism onto its image.

(iv) $i: D \to \mathcal{U}$ is an order-isomorphism iff D has a top.

(v) $i: D \to \mathcal{U}$ preserves all infima.

In particular, items (ii)–(v) are valid for *any* universal pair for D.

PROOF. By [Win 1988], $\mathcal{U}$ is an algebraic lattice that is order-generated by complete primes, and $\mathcal{U}$ is therefore distributive. If $F \subseteq \mathrm{Pr}(D)$ is finite, then $\downarrow(F)$ is a finite set, so every $\downarrow(F) \in \mathrm{K}(\mathcal{U})$ has only finitely many elements below it. Hence, $\mathcal{U}$ is a dI-domain. For $x \in D$, the set D_x is a lower set in $\mathrm{Pr}(D)$. Hence, i is well-defined. Now,

$$(18) \qquad D_{\sqcup X} = \bigcup_{x \in X} D_x$$

implies that i preserves all existing suprema. Also, $\mathrm{Pr}(D) \cong \{D_p \mid p \in \mathrm{Pr}(D)\} = i(\mathrm{Pr}(D)) = \mathrm{Pr}(\mathcal{U})$ can be easily seen. If E is a complete lattice and $f: D \to E$ a map preserving all existing suprema, then

$$(19) \qquad \bar{f}(L) := \bigsqcup_{p \in L} f(p)$$

is well-defined, as E is a complete lattice. Clearly, $\bar{f}: \mathcal{U} \to E$ preserves all suprema, and $\bar{f} \circ i = f$ is immediate. As $\mathrm{Pr}(\mathcal{U}) = i(\mathrm{Pr}(D))$ such a function $\bar{f}$ is unique. This proves (i) and (ii).

But (iii) is true because $x \leq y$ and $D_x \subseteq D_y$ are equivalent relations. For (iv), the only interesting case is when D has a top. Then, D is a complete lattice and $\mathrm{id}_D : D \to D$ has a unique extension $\overline{\mathrm{id}_D}: \mathcal{U} \to D$ such that $\overline{\mathrm{id}_D} \circ i = \mathrm{id}_D$. The maps

$$(20) \qquad \mathrm{id}_{\mathcal{U}}, i \circ \overline{\mathrm{id}_D}: \mathcal{U} \to E$$

preserve all suprema and are extensions of $i: D \to \mathcal{U}$. By uniqueness, $\mathrm{id}_{\mathcal{U}} = i \circ \overline{\mathrm{id}_D}$ holds. Finally, (v) is a consequence of

$$(21) \qquad D_{\sqcap X} = \bigcap_{x \in X} D_x.$$

□

13 Lemma. Let D and E be dI-domains, where $(\mathcal{U}_\mathcal{D}, i_D)$ and $(\mathcal{U}_\mathcal{E}, i_E)$ are their respective universal pairs. Then $\mathrm{Pr}(D) \cong \mathrm{Pr}(E)$ implies $\mathcal{U}_\mathcal{D} \cong \mathcal{U}_\mathcal{E}$.

PROOF. Let $\varphi: \mathrm{Pr}(E) \to \mathrm{Pr}(D)$ be an order-isomorphism. Define $i: E \to \mathcal{U}_\mathcal{D}$ by

$$(22) \qquad i(x) := \bigsqcup_{p \in E_x} (i_D \circ \varphi)(p).$$

Then, it is straightforward to show that $(\mathcal{U}_\mathcal{D}, i)$ is a universal pair for E, which implies $\mathcal{U}_\mathcal{D} \cong \mathcal{U}_\mathcal{E}$. □

It can also be shown that $\mathcal{U}_\mathcal{D} \cong \mathcal{U}_\mathcal{E}$ implies $\mathrm{Pr}(D) \cong \mathrm{Pr}(E)$.

PROOF OF THEOREM 11. By Lemmas 12 and 13, it suffices to show that $\mathrm{Pr}(D) \cong \mathrm{Pr}(\mathrm{Proj}(D))$, for $\mathrm{Proj}(D)$ is a dI-domain and has id_D as a top element. Hence, it suffices to show that

$$(23) \qquad \mathrm{Pr}(\mathrm{Proj}(D)) = \{\lambda x.x \sqcap c \mid c \in \mathrm{Pr}(D)\}.$$

For $p \in \mathrm{Proj}(D)$, $x \leq p(y)$ implies $p(x) = p(y) \sqcap x = x$ as $p \sqsubseteq \mathrm{id}_D$. So $\mathrm{im}(p)$ is a non-empty Scott-closed set of D. Moreover, if $Y \subseteq \mathrm{Proj}(D)$ is given, then

$$(24) \qquad N := \{\sqcup F \mid F \subseteq \bigcup_{q \in Y} \mathrm{K}(\mathrm{im}(q)) \text{ and } F \text{ finite }\}$$

is a normal lower subset of $\mathrm{K}(D)$, as D is distributive.

Define $p_N(x) := \sqcup(\downarrow(x) \cap N)$. Then, $p_N \in [D \xrightarrow{\mathrm{pr}} D] = \mathrm{F}_{\mathrm{pr}}(D)$ follows, as D is projection-stable. But $\mathrm{K}(\mathrm{im}(p_N)) = N$ implies that $\mathrm{im}(p_N)$ is the Scott-closure of the lower set N. For $x \leq y$ in D, we get

$$p_N(x) = p_N(x \sqcap y) = \sqcup(\downarrow(x \sqcap y) \cap N) = \sqcup((\downarrow(y) \cap N)) \sqcap x,$$

which implies $p_N \sqsubseteq \mathrm{id}_D$. For $x, y \leq z$, we have $p_N(x \sqcap y) = p_N(x) \sqcap p_N(y)$, using the fact that $n_1 \sqcap n_2 \in N$ for $n_1, n_2 \in N$. But then $p_N \in \mathrm{Proj}(D)$ follows. For $q \in Y$, $\mathrm{K}(\mathrm{im}(q)) \subseteq \mathrm{K}(\mathrm{im}(p_N))$ implies $q \leq p_N$, as D is projection-stable. By (16), this means $q \sqsubseteq p_N$, as we have $q, p_N \sqsubseteq \mathrm{id}_D$. So p_N is an upper bound of Y in $\mathrm{Proj}(D)$.

Let $h \in \mathrm{Proj}(D)$ be another upper bound of Y in $\mathrm{Proj}(D)$. For $k \in \mathrm{K}(\mathrm{im}(p_N))$, we have $k = \sqcup F$, where $F \subseteq \cup\{\mathrm{K}(\mathrm{im}(q)) \mid q \in Y\}$ is finite. For $l \in F$ there exists some $q \in Y$ with $l = q(l) \leq h(l)$. Thus, $k = \sqcup F \leq \sqcup h(F) \leq h(\sqcup F) = h(k)$ implies $k \in \mathrm{K}(\mathrm{im}(h))$. Again, $p_N \leq h$ follows, as D is projection-stable. By (16), we get $p_N \sqsubseteq h$ and p_N is the supremum of Y in $\mathrm{Proj}(D)$.

If $\lambda x.x \sqcap c \leq p_N$ for some $c \in \mathrm{Pr}(D)$, then $c = \sqcup(\downarrow(c) \cap N)$ implies $c \in N$, so $c = \sqcup F$ for some finite $F \subseteq \cup\{\mathrm{K}(\mathrm{im}(q)) \mid q \in Y\}$. As c is prime, we have $c \in F$, so $c \in \mathrm{K}(\mathrm{im}(q))$ for some $q \in Y$. Thus, $\mathrm{im}(\lambda x.x \sqcap c) = \downarrow(c) \subseteq \mathrm{K}(\mathrm{im}(q))$ implies $\lambda x.x. \sqcap c \leq q$, and again $\lambda x.x \sqcap c \sqsubseteq q$ follows. Therefore, $\lambda x.x \sqcap c \in \mathrm{Pr}(\mathrm{Proj}(D))$ has been proven.

Conversely, let $p^* \in \mathrm{Pr}(\mathrm{Proj}(D))$ be given. Set

$$(25) \qquad \tilde{p} := \bigsqcup_{c \in \mathrm{Pr}(D) \cap \mathrm{im}(p^*)} \lambda x.x \sqcap c.$$

We are done, if $p^* = \tilde{p}$. Now, $\tilde{p} \leq p^*$ is clear. For $p^* \leq \tilde{p}$ it suffices to show that $\mathrm{K}(\mathrm{im}(p^*)) \subseteq \mathrm{K}(\mathrm{im}(\tilde{p}))$, as D is projection-stable. Let $k \in \mathrm{K}(\mathrm{im}(p^*))$ be given. For

$$(26) \qquad \bar{p} := \bigsqcup_{c \in D_k} \lambda x.x \sqcap c,$$

we have $\bar{p} \leq \tilde{p}$. Now, $\bar{p}(k) \geq \sqcup\{k \sqcap c \mid c \in D_k\} = k \sqcap (\sqcup D_k) = k$ follows as D is distributive and satisfies the axiom (Pr). Hence, $k \in \mathrm{K}(\mathrm{im}(\bar{p})) \subseteq \mathrm{K}(\mathrm{im}(\tilde{p}))$ does it. □

14 Corollary. Let D be a dI-domain. Then Proj(D) is isomorphic to the Hoare power domain of the ideal completion of the poset of complete primes of D:

$$\mathrm{Proj}(D) \cong \wp_H(\mathrm{Id}(\mathrm{Pr}(D))).$$

In particular, Proj(D) is a completely distributive bialgebraic lattice.

PROOF. Let $E := \mathrm{Id}(\mathrm{Pr}(D))$ be the ideal completion of the poset $\mathrm{Pr}(D)$ and $\wp_H(E)$ the Hoare power domain thereof. By [Win 1985], we have $\mathcal{L}(\mathrm{K}(E)) \cong \wp_H(E)$. Now, $\mathrm{K}(E) \cong \mathrm{Pr}(\wp_H(E))$ implies $\mathrm{Pr}(D) \cong \mathrm{Pr}(\wp_H(E))$, so $\mathrm{Proj}(D) \cong \mathcal{U}_D \cong \mathcal{U}_{\wp_H(E)} \cong \wp_H(E)$ follows, as $\wp_H(E)$ is a dI-domain and has E as top element. By [Mis et.al. 1991], Hoare power domains of algebraic dcpos are completely distributive and bialgebraic. □

5 Conclusion

We have outlined what needs to be done if one wants a cartesian closed category of algebraic dcpos with bottom and Scott-continuous maps as arrows which is also closed under the operator $D \mapsto [D \xrightarrow{\mathrm{pr}} D]$. More importantly, we have shown that the cartesian closed category of dI-domains with stable maps in the stable order as the function space is closed under its projection space operator. Using Berardi's results, it is also closed under the retraction space and images of stable retractions. This is in striking contrast to the situation in $\mathrm{ALG}_\perp$. The reader may have realized that Proj(D) has an isomorphic copy in form of the completely distributive algebraic lattice of lower normal subsets of K(D)—or of normal Scott-closed sets of D—in the inclusion order. In fact, it can be easily seen that $p \in [D \xrightarrow{\mathrm{pr}} D]$ with $p \sqsubseteq \mathrm{id}_D$ is a stable function, and that the order on Proj(D) is the pointwise one.

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7 References

erardi S.
1991] *Retractions on dI-domains as a model for type:type*, in: *Information and Computation*, vol.**94** (**1991**), pp.204–231

erry G.
1978] *Stable models of typed λ-caculi*, in: *Fifth International Colloquium on Automata, Languages and Programs*, Springer Verlag, Lecture Notes in Computer Science, vol. **62** (**1978**), pp.72–89

Cantor G.
[1884] *De la puissance des ensembles parfaits de points*, Acta Math. **4** (**1884**), pp.381–392

Coquand T., C. Gunter and G. Winskel
[1989] *DI-Domains as a Model of Polymorphism*, in: *Third International Conference on the Mathematical Foundations of Programming Semantics*, Springer Verlag, Lecture Notes in Computer Science, vol. **298** (**1989**), pp.344–363

Gierz G., K.H.Hofmann, K.Keimel, J.Lawson, M.W. Mislove and D.Scott
[1980] *A Compendium of Continuous Lattices*, Springer Verlag, New York, 1980

Gunter C.
[1985] *Profinite Solutions For Recursive Domain Equations*, Doctortal Dissertation, University of Wisconsin at Madison, 1985

Gunter C., P.Mosses and D.Scott
[1990] *Semantic Domains And Denotational Semantics*, in: *Handbook of Theoretical Computer Science*, vol.**2**, Jan van Leeuwen (ed.), North-Holland, Amsterdam, 1990

Hofmann K.H., M.W. Mislove, and A. Stralka
[1973] *Dimension Raising Maps in Topological Algebra*, Mathematische Zeitschrift **135**, pp.1–36, Springer Verlag, New York, 1973

Hofmann K.H. and M.W. Mislove
[1977] *The Lattice of Kernel Operators and Topological Algebra*, Mathematische Zeitschrift **154**, pp.175–188, Springer Verlag, New York, 1977

Huth M.
[1990] *Projection-stable and Zero Dimensional Domains*, Doctoral Dissertation, Tulane University 1990

Huth M.
[1991] *dI-domains and the Hoare power domain*, preprint, Tulane University 1991

Jung A.
[1989] *Cartesian Closed Categories of Domains*, CWI Tract **66**, Amsterdam 1989, 110pp.

Mislove M.W. and F.J. Oles
[1991] *A Simple Language Supporting Angelic Nondeterminism and Parallel Composition*, these proceedings

Scott D.
[1982] *Domains for Denotational Semantics*, in: ICALP 82, M.Nielsen and E.M.Schmidt (eds.), Springer LNCS **140** (**1982**), pp.577–613

Smyth M.B.
[1983] *The Largest Cartesian Closed Category of Domains*, TCS **27** (**1983**), pp.109–119

Winskel G.
[1985] *On Power Domains And Modality*, TCS **36** (**1985**), pp.127–137, North-Holland
[1988] *An Introduction to Event Structures*, in: *Linear Time, Branching Time and Partial Order in Logics and Models for Concurrency*, Lecture Notes in Computer Science **354** (**1988**), pp.364–397, Springer Verlag