Artistic box trees

Cameron Browne

SWiSHzone.com Pty Ltd

The Basement, 33 Ewell St, Balmain, Australia, 2041

Cameron.Browne@swishzone.com

Abstract

This short note describes a development of the traditional Pythagorean tree fractal to produce interesting three-dimensional structures based on cubes rather than squares. Adding a 90˚ rotation at each bifurcation encourages three-dimensional growth, creating increasingly artistic shapes as the branching angle decreases.

1. Pythagorean box trees

The Pythagorean tree is a plane fractal constructed with squares, such that each triplet of touching squares encloses a right-angled triangle [1]. The tree shown in Figure 1 is described as P_{45} as the smallest internal angle of each such triangle is θ =45.

Figure 2 shows a Pythagorean tree P_{30} in which each square has been extruded to a cube to produce a *box tree*. The left column of figures show the first three steps in this construction; starting with a single cube (top left), two subcubes are added such that the interior gap forms a 30° right-angled triangle when viewed in front (middle left), then two subcubes are similarly added to these (bottom left), and so on. The figure on the right shows the construction after several iterations, which although made of three-dimensional elements is still essentially planar in nature.

2. Branch rotation

Figure 3 shows a box tree based on P_{45} in which the branching direction is rotated 90° with each iteration, allowing a more truly three-dimensional spread of the canopy. This structure displays *pmm symmetry* (perpendicular axes of reflection) when viewed from directly above or below.

An interesting by-product of branch rotation is that the box tree of P_{45} with rotation is no longer self-intersecting, although both the original P_{45} tree and the box tree based upon P_{45} without rotation are.

Figure 4 shows a box tree based on P_{30} with a similar branch rotation scheme. The resulting structure lacks symmetry and has a rather random natural look, however it is exactly reconstructible from its base angle as are all the figures in this paper.

As the base angle decreases, Pythagorean box trees with branch rotation grow taller and some interesting shapes emerge. For instance, Figure 5 shows the box tree based on P_{20} with branch rotation that forms a gradually sweeping spiral as it grows, viewed from above. Similarly, Figure 6 shows a stereo pair (cross-eyed stereo) of an elegant P_{15} box tree with branch rotation.

3. Conclusion

This paper describes a development of the Pythagorean tree to produce interesting three-dimensional structures formed by cubes, especially with the incorporation of a 90˚ rotation at each bifurcation point. It might be interesting to investigate whether all such trees with branch rotation are non-self-intersecting.

References

[1]   Peitgen, H., Jürgens, H. and Saupe, D. *Chaos and Fractals: New Frontiers of Science*, Springer, New York, 1992.
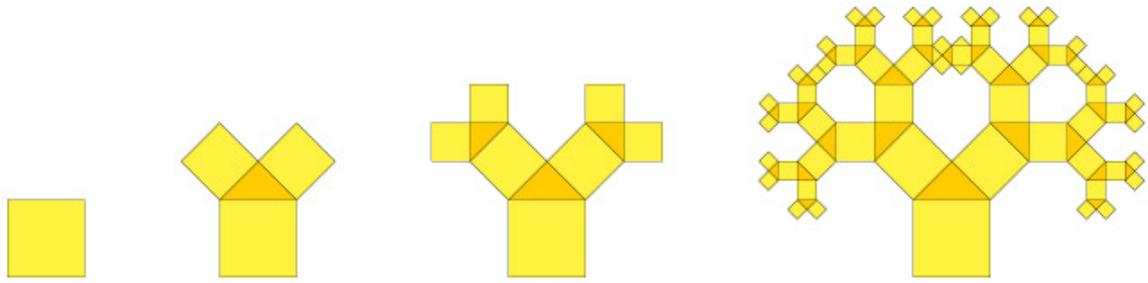
Figures

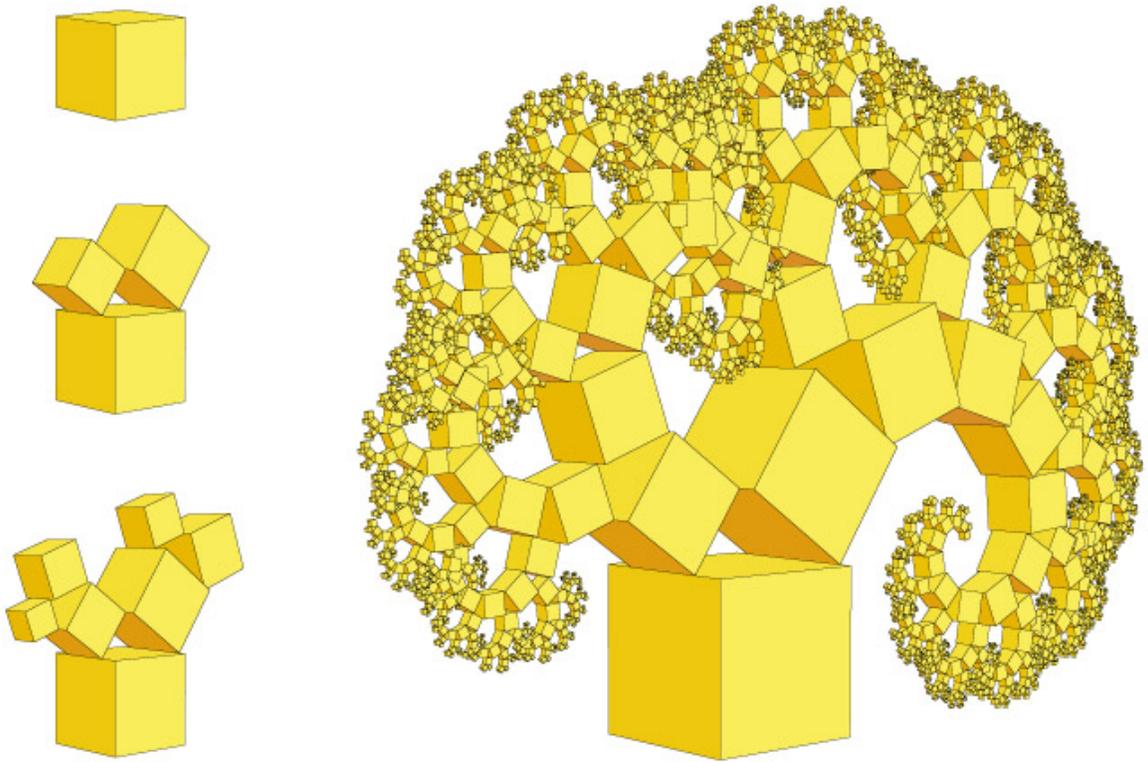**Figure 1.** First four stages of the Pythagorean tree P_{45}.
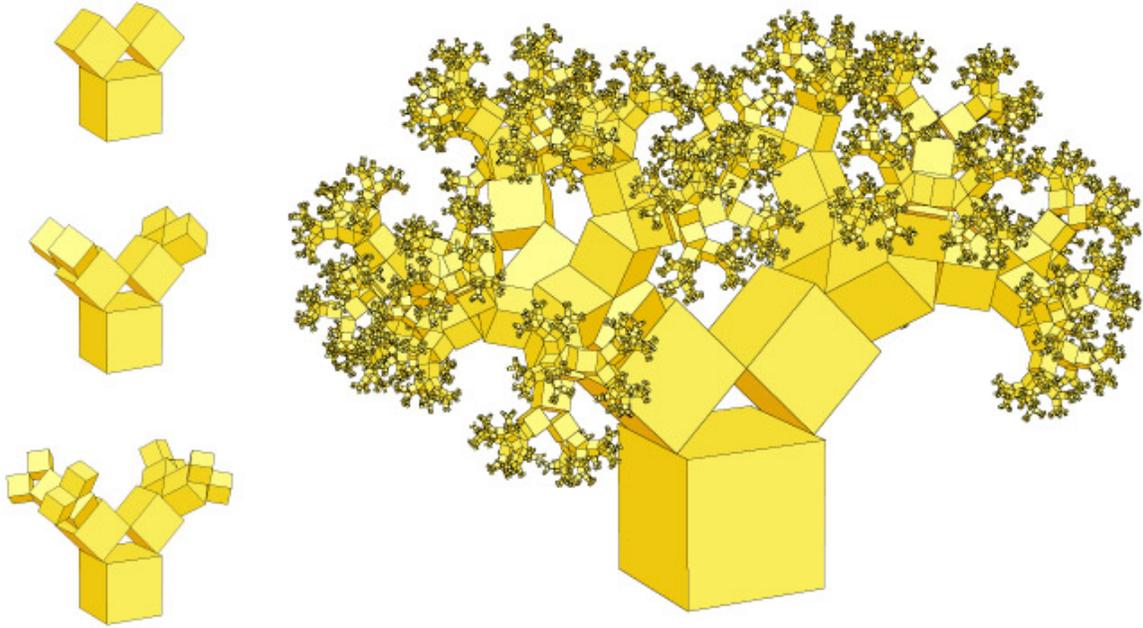

Figure 2. A box tree extruded from P_{30}.

Figure 3. A box tree based upon P_{45} with branch rotation.
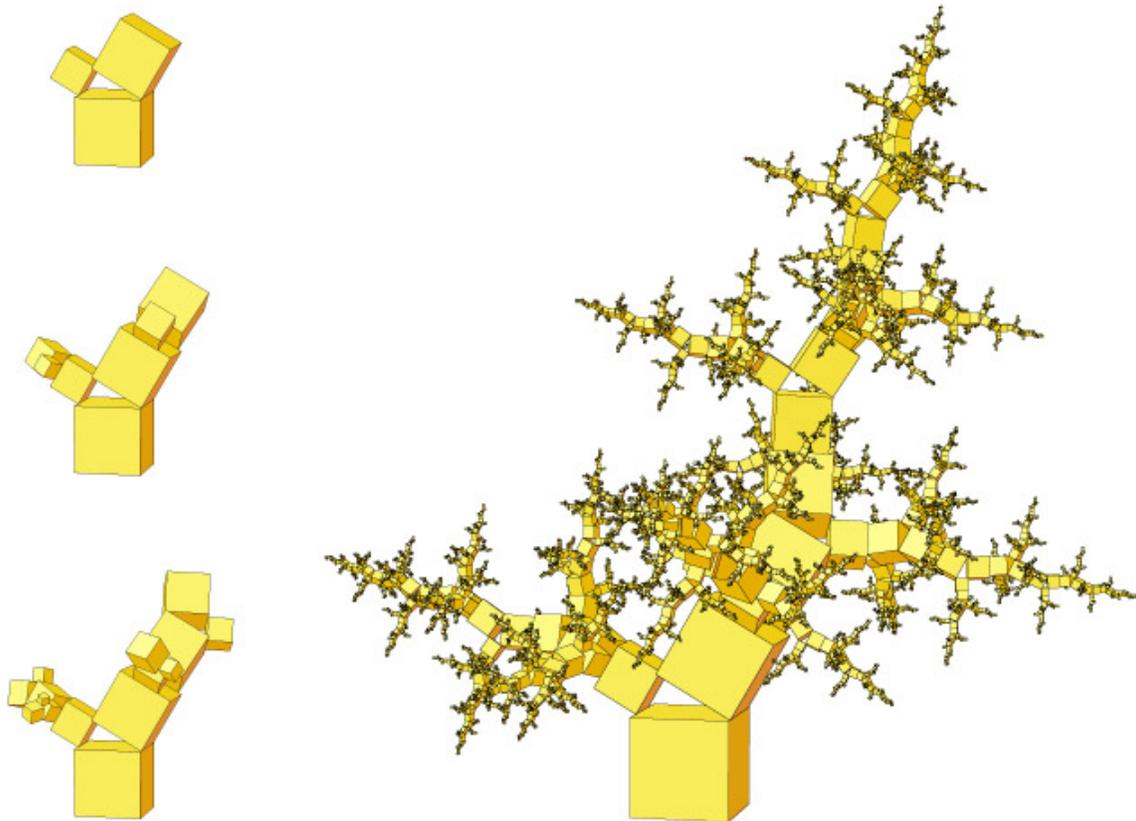

Figure 4. P_{30} box tree with branch rotation.
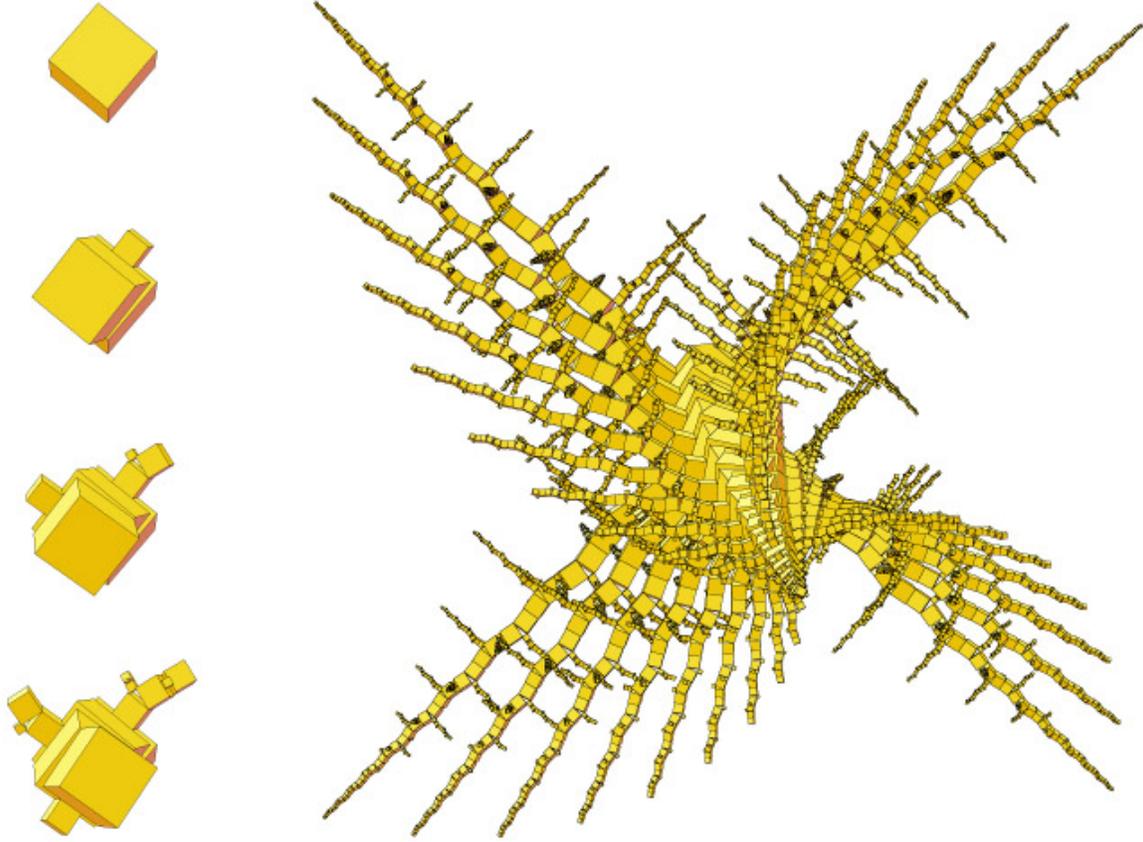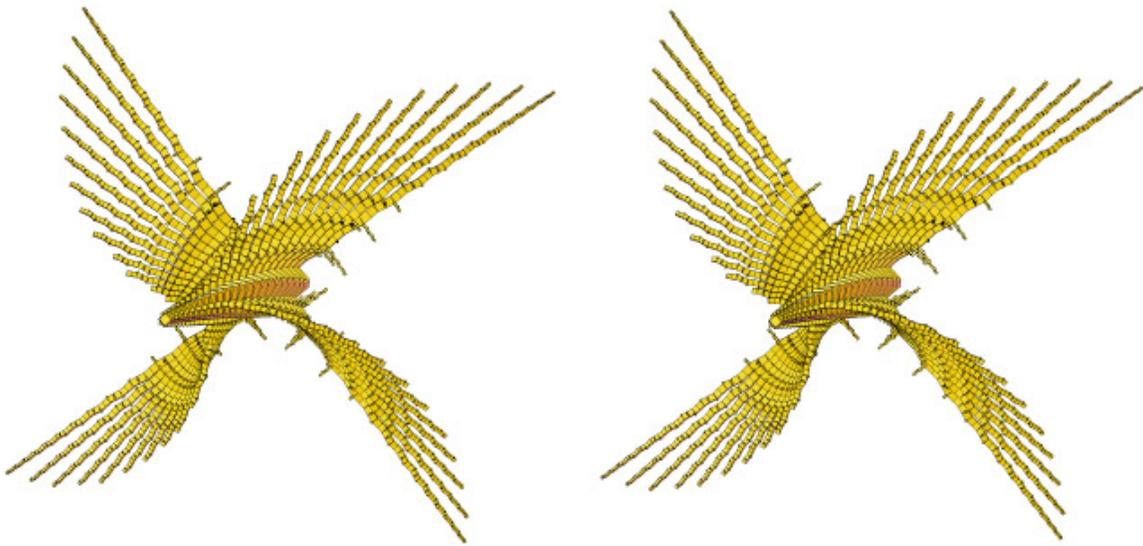
Figure 5. P_{20} box tree with branch rotation.

Figure 6. Stereo pair of a P_{15} box tree with branch rotation.