Mean-field approximations for performance models with deterministic timeouts

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Introduction
We show how popular techniques for analysing massively parallel stochastic systems, for instance mean-field [1] and fluid-analysis [2], can be extended directly to allow deterministically as well as exponentially timed transitions.

Deterministically-timed events
Ubiquitous in real-world computer and communication systems, for example:
- Timeouts in networking protocols;
- Impatient or re-queuing customers;
- Networks with fixed-length packets;
- Time to reset/reboot a server;
- Other isolated or predictable tasks which always take the same amount of time.

Analysing the underlying process
Generalised semi-Markov process (GSMP) [3], i.e. discrete state-space is non-Markovian since future evolution may depend on elapsed time for deterministically-timed transitions.
- General approaches require solution of multi-dimensional PDEs [4] or Fredholm equations [5];
- Number of equations grows proportionally to the state-space size — does not scale to systems with many interacting entities;
- Other approaches (e.g. [6]) impose very significant structural restrictions on the enabling of non-exponential transitions.

Alternatively, deterministic durations can be approximated by k-stage Efrang distributions. This is also often impractical since k must be large for an accurate result.

Mean-field approach
Let \( S = (x, y, z, u, v) \) be the discrete-state stochastic process counting the number of nodes in each state. For a state \( S = (x, y, z, u, v) \), the notation \( s_{x,y,z,u,v}^{-1} \) for \( x, y, z, u, v \), represents the state \( (x - 1, y, z, u + 1, v) \). By considering what can happen in \([t, t+\delta t]\), we can show:

Other transients with nasty joint dependence on the past

Two problems: number of equations (state-space explosion) and intractable joint dependence on the past in the extra terms.

A solution: multiply each equation by node count and sum over all states to obtain equation for node-count expectations. Then apply usual mean-field approximation of delay-differential equations (DDEs).

Two problems: approximation and intractable joint dependence on the past.

DDE approx. (solid line) compared with actual means for \( N = 20, 50, 100 \) (dashed lines)

Conclusion & future work
- DDEs can be used for tractable mean-field analysis of systems with deterministic timeouts.
- Future work includes theoretical convergence results and error bounds.
- Also, possible extensions to higher-order moments (cf. [2]), reward measures (cf. [7]) and more general distributions.

References

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