Abstract

We introduce an approximation algorithm for the evaluation of networks of fluid queues. Such models can be used to describe the generation and storage of renewable energy.

We discuss how our algorithm would be applied to an example where the approximation performs very well, and note a modification to the model which would result in a poorer result.

Fluid queue

A fluid queue consists of a continuous time Markov chain on a set of rates and a buffer with a fixed service rate. The state of the CTMC $J_t$ (describing the weather, say) determines the input rate to the fluid buffer. Once fluid arrives in the buffer it is served immediately at rate $\mu$ (here set to 10). Any fluid that cannot be served immediately accumulates in the buffer.

Output process

The output process of a fluid queue $Y_t$ is given by

$$Y_t = \begin{cases} \mu & \text{if } X_t > 0 \\ \lambda & \text{if } X_t = 0. \end{cases}$$

The busy period is the period of time measured from the instant a buffer first fills to the time it empties – the time for which the output process is $\mu$.

Renewable energy sources are modeled as Markov modulated sources of power feeding into local storage buffers at nodes 1 and 2. (Capacity factors are typically 20–40% for most renewable sources, so the chosen parameters are accurate on average.)

At the first stage we can compute quantities in the model exactly. In order to calculate the inputs to the central storage buffer at node 3 we need to approximate the output processes of nodes 1 and 2 by a continuous time Markov chain-modulated process. (The busy periods of a fluid queue do not have a phase type distribution.) We use a least squares algorithm to match moments of the busy period and our approximation.

We calculate what happens at node 3 using the approximation processes. The busy period at node 3 in the network below represents the time period for which power demand is satisfied.

The approximation performs well and the estimate of the busy period matches the actual value (obtained from simulation) to three significant figures as 2.26.

Inputs to node 3

We approximate the non-exponential transition (see left) by a phase type transition (see right) with matching moments using least squares minimization.

To apply this approximation method to a general fluid queue we need to include more than one on state (it might be that two on states can only happen through two distinct paths in the state space) and how to better compute joint state spaces to include both dependencies and to keep the state space size manageable.

Dependencies

If we change the model and replace sources 2 and 3 by a joint source with rate 60 this would increase the busy period to 3.00, while our approximation (currently ignoring these dependencies) would remain the same as 2.26. Clearly dependencies need to be included in future work.

Conclusions and future work

We have introduced an approximation algorithm and shown it to perform well in a small example describing the delivery of renewable power to consumers. To model networks more accurately we need to include information about correlations and model losses explicitly (these are significant as most energy storage is around 75% efficient).