Fluid computation of the performance–energy trade-off in large scale Markov models

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Introduction

We show how fluid analysis techniques can be used to assist providers of large scale systems with selecting resources and deployment strategies that guarantee service level agreements and minimise energy consumption.

Example

Simple cloud computing model with 2 classes of jobs, 2 classes of servers

State space

The system is specified in a simple stochastic process algebra with channels capturing resource allocation:

\begin{align*}
\text{Sch} & \xrightarrow{\text{assign}_{x \leftarrow \text{s}}x} \text{Sch} \\
\text{Job} & \xrightarrow{\text{assign}_{x \leftarrow \text{s}}x} \text{Job} \\
\text{Server} & \xrightarrow{\text{assign}_{x \leftarrow \text{s}}x} \text{Server} \\
\text{Proc} & \xrightarrow{\text{assign}_{x \leftarrow \text{s}}x} \text{Proc}
\end{align*}

This extends to multi-processor servers.

Slower system can break the SLA

Formal specification

System specified in a simple stochastic process algebra with channels capturing resource allocation:

\begin{align*}
\text{request get}_x & \xrightarrow{} \text{Sch} \\
\text{request init}_x & \xrightarrow{} \text{Sch} \\
\text{assign forward}_x & \xrightarrow{} \text{Sch} \\
\text{assign get}_x & \xrightarrow{} \text{Sch} \\
\text{assign}_x & \xrightarrow{} \text{Server}
\end{align*}

The Sch component decides which jobs get deployed on which servers. A job can use the assigned server for processing:

\begin{align*}
\text{Job} & \xrightarrow{} \text{proc on}_x \text{Server}
\end{align*}

Populations

Population Continuous Time Markov Chains (CTMCs) are STMCs where the state space consists of states \( x = (x_1, \ldots, x_n) \in \mathbb{N}^n \). Events are specified as

\( R(x) : dX_1 + \cdots + dX_n \rightarrow iX_1 + \cdots + iX_n \)

State \( x \) such that \( x_i \geq d_i \) for \( i = 1, \ldots, n \) goes to state \( x - d + i \) with rate \( R_i(x) \).

Fluid analysis

The structure of PCTMCs is suitable for deriving ODEs that approximate the evolution of means and higher joint moments of individual populations over time [1], i.e. with solutions

\[ \frac{d}{dt} E(x(t)) = \cdots \]

The solutions are usually accurate [2]. Moments can be used to compute further derived metrics, such as the probabilities needed in SLAs [3].

Energy consumption

\[ S_i(t) \text{ is the count of components in state } i. \text{ The total component-time spent in state } i \in \sum_{t=0}^{T} S_i(t). \]

Energy consumption can be modelled as a linear combination of the total component-times spent in each state. [4] shows how to derive ODEs that approximate moments of these accumulated measures.

Conclusion & Future work

- Fluid analysis provides access to SLA and energy consumption metrics of systems with extremely large state spaces.
- Low computational cost can be used to explore various system configurations.
- We have a prototype implementation of the techniques [5].
- We are investigating rigorous optimisation algorithms that use the ODE structure to guarantee global minima.

References


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