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Fractal board games

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Abstract

Connection games are a recent genre of abstract board games with some interesting geometrical properties. We introduce a recursive metarule with which existing connection games can be expanded into fractal-like variants with recursively defined play. Computer graphics are used to extrapolate a theoretical game played on a continuous potential field to a similar set of games that are truly fractal in nature.

Keywords: Fractal; Abstract board game; Connection game; Hex; Y

1. Connection games

Connection games are a family of abstract board games in which players strive to complete a particular type of connection with their pieces. The inherent nature of connection allows these games to be quite complex despite having extremely simple rules. The basic concept of connection will be familiar to most players, making these games intuitive and easy to grasp though usually hard to play well.

Fig. 1 shows two famous connection games, Hex and Y, and a recent one called Quax. Each game is played on a different board, but their rules are similar and very simple; players take turns placing a piece of their color on an empty cell, and win by completing a path of their pieces between either their sides of the board (Hex and Quax) or all sides of the board (Y). Each of the games shown in Fig. 1 has been won by White.

An attractive feature of Hex and Y is that there can be no ties; exactly one player must win. This is due to the trivalent nature of the hexagonal tiling, which unlike the square grid has no diagonal neighbors (explained in detail in [1]). Quax, invented by New Zealand mathematician Bill Taylor in 2000, has an additional rule that players may place a bridge between diagonal pieces of their color, to avoid deadlocks on the square grid [2].

Connection games are described as *variable geometry* games, which means that the size and shape of the connection does not matter; it is the mere fact of connection that counts. For this reason, connection games can be played on a wide variety of board designs, and tend to scale well to different board sizes without rule changes.

2. Quadrant games

The idea of recursively-defined connection games has occurred to a number of people, but Steven Meyers, a game designer from Cincinnati, USA, was the first to suggest a workable set of rules with his game Quadrant Hex in 2000 [3].

Fig. 2 shows the previously mentioned game of Hex realized as a game of Quadrant Hex. The main board (called the *supergame*) is split into four quarter-sized boards (called the *subgames*) and a game of Hex is played in each according to the same rules. Players still only place one piece per turn on the supergame; each move is implied on the relevant subgame. In other

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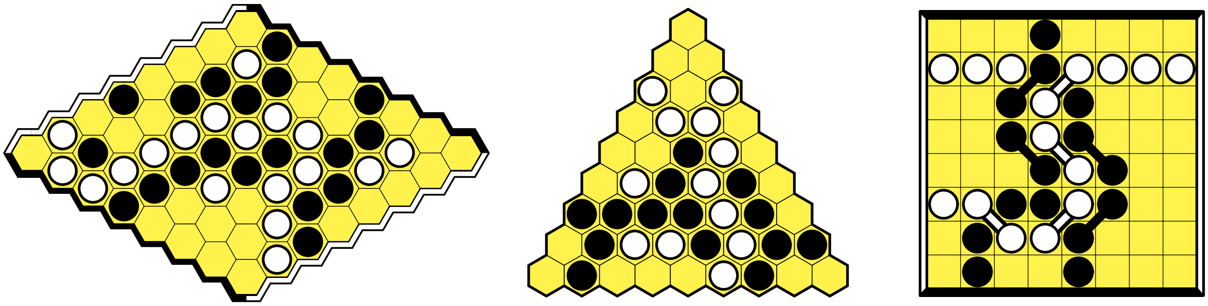

Fig. 1. Games of Hex, Y and Quax, each won by White.

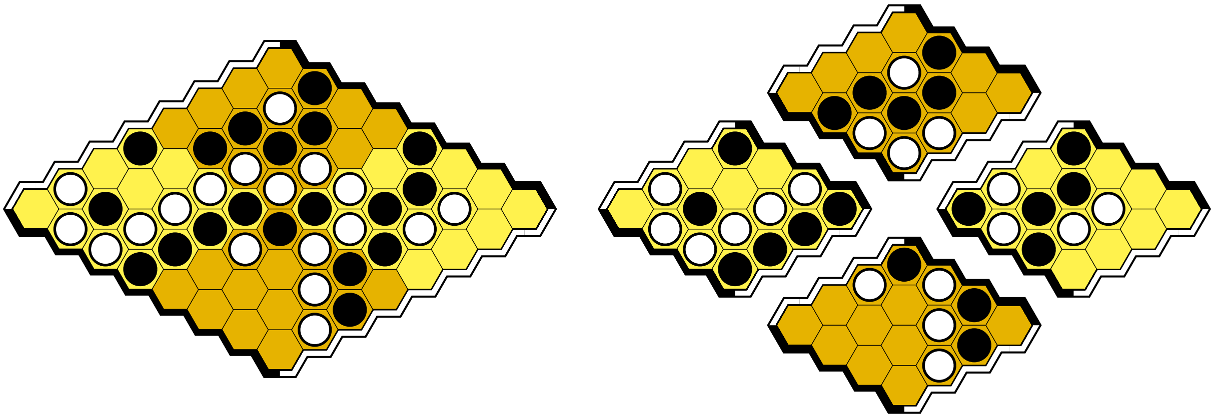

Fig. 2. A game of Quadrant Hex won by Black (3–1).

words, the single piece placed each turn is effectively placed once on every level of subdivision.

Quadrant Hex is best played with an even number of cells along each side of the supergame to avoid overlap between subgames. The colored board regions serve no purpose except to make the subdivision explicit to the players; implicit subdivision on a uniform board is perfectly legitimate but is more difficult for the players.

Quadrant Hex is won by the player who wins the majority of the five component games. Play does not necessarily stop as soon as the supergame is won, and in fact winning the supergame has no more bearing on the outcome than winning a subgame. This is shown in Fig. 2, where Black has won the overall match by winning three component games (all subgames) while White has only won one component game (the supergame). Note that it is not necessary to play all component games to completion to determine the winner. Like Hex, exactly one player must win in Quadrant Hex.

This recursive subdivision principle constitutes a metarule that can be applied to most connection games played on a regularly tiled board whose shape constitutes a *rep-tile* (a polygon that can be dissected into smaller copies of itself [4]). Quadrant Hex should technically be called Quarter Hex, but the term *quadrant* will continue to be used for historical reasons.

Fig. 3 shows the previously mentioned game of Y realized as a game of Quadrant Y, using the same recursive principle. Again, Black wins the overall match by winning three subgames, even though White has won the supergame and one subgame.

Note that the central Quadrant Y subgame is surrounded and shares an edge with each of the three adjacent subgames, so that a piece on the shared edge of one subgame is also implied on the other. This is necessary if all four subgames are to be of equal size, and means that the Quadrant Y supergame *must* have an odd number of cells along each side.

Fig. 4 shows the previously mentioned game of Quax realized as a game of Quadrant Quax. The overall match has been won by White, who has won the supergame and two subgames. Black has only won two subgames.

Note that bridges between pieces in adjacent subgames do not survive the subdivision process. Luckily this does

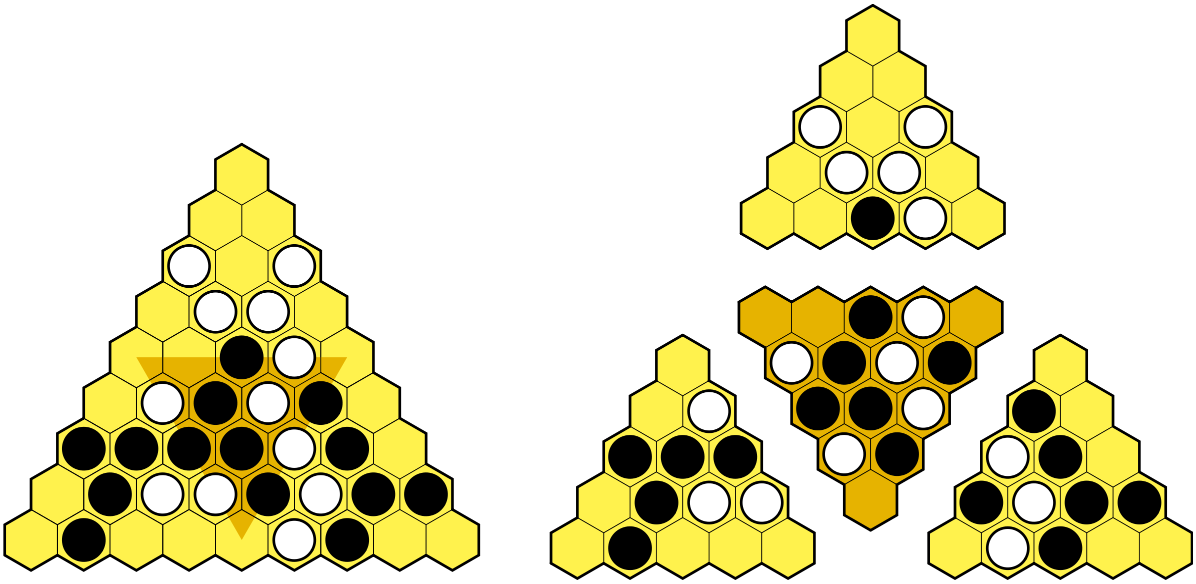

Fig. 3. A game of Quadrant Y won by Black (3–2).

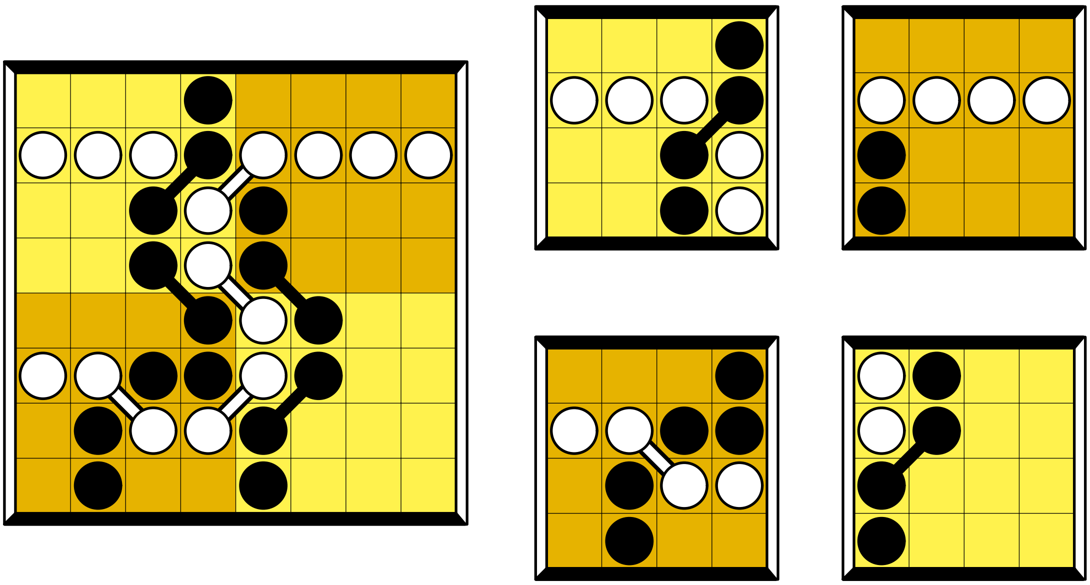

Fig. 4. A game of Quadrant Quax won by White (3–2).

not affect the outcome in Quadrant Quax, but points to a potential problem with games that require special rules to work properly. For instance, some games may involve the capture of groups with no liberties to clear up board space. However, groups with no liberties within a subgame may have liberties in the supergame, creating a contradiction that makes such games unsuitable for recursion.

Connection games on the hexagonal grid generally do not require such special rules. These make ideal candidates for recursive games for a number of reasons:

- They scale well to half-size reductions.
- The variable-geometry nature of connection makes it equally viable at all levels.

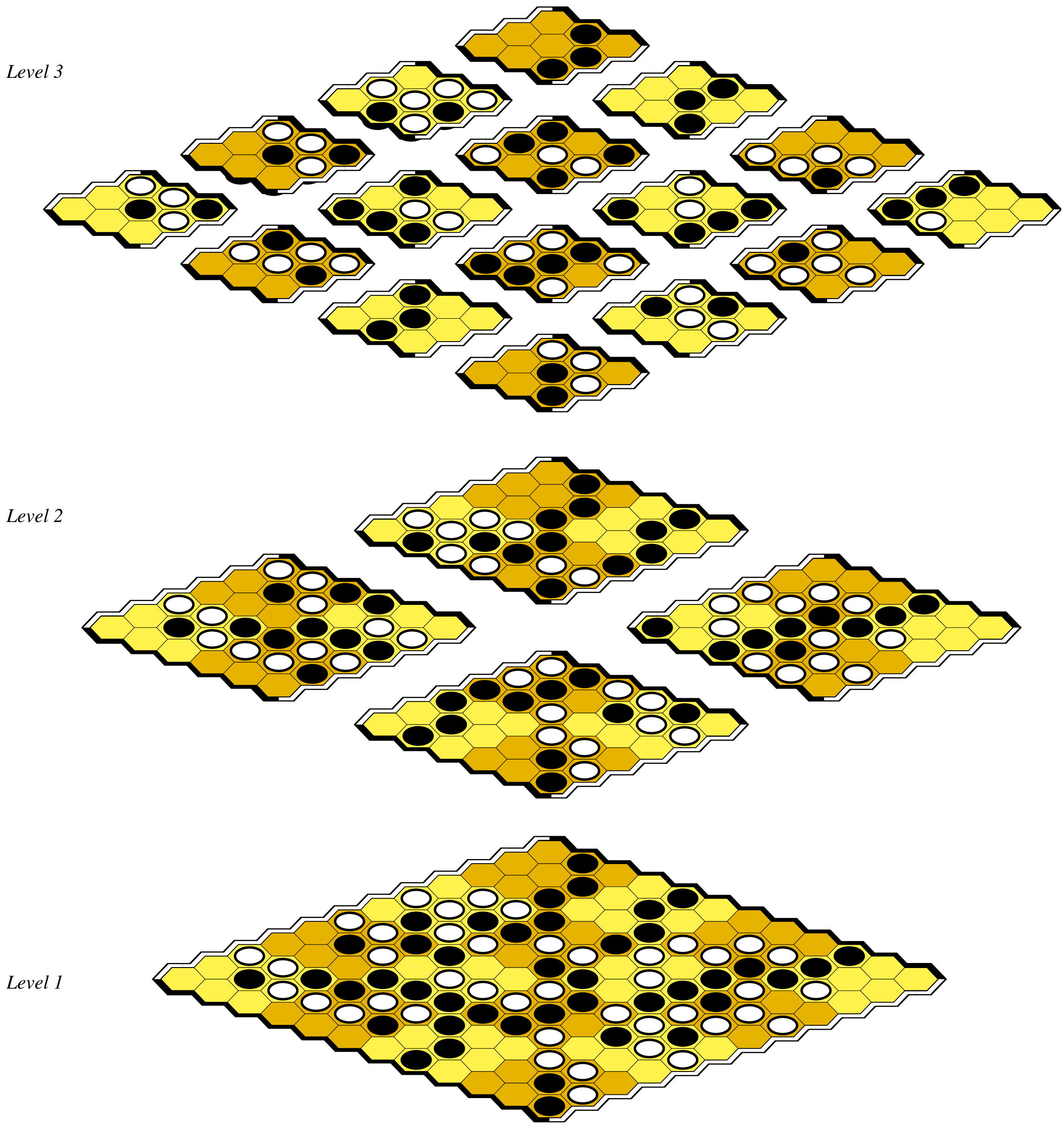

Fig. 5. The 21 component games of level 3 Quadrant Hex. Next move to win (either color).

- Connection provides a unified theme across the entire game, and there are no special pieces or conditions to complicate subdivision. For instance, subdividing a game of Chess would not work very well, as the three subgames that did not contain a player's King would immediately be implied losses for that player.

3. Fractal-like games

The recursive games described above are fractal-like as they display self-similarity at all levels [4]. In other words, the same game is played using the same rules at each level. However, it can be argued that these games are not truly fractal as only one level of subdivision occurs, and fractals are generally assumed to possess infinite detail.

While it is possible to further subdivide larger boards, there will be a hard limit imposed by the discrete board size. For instance, Fig. 5 shows a 12×12 game of Hex in which each subgame recursively becomes its own game of Quadrant Hex. This board cannot be subdivided any further, and yields a total of $1+4+16=21$ component games.

Levels are labeled $1 \ldots n$, where 1 describes the supergame and n describes the maximum level of subdivision. Defining a variable d for the degree of subdivision ($d=4$ for quadrant games), then the total number of component games $T_{d,n}$ is given by

$$T_{d,n} = \sum_{i=0}^{n-1} d^i. \quad (1)$$

One of the key features of recursive games is that play flows between levels, allowing players to pose devious simultaneous threats in both supergame and subgame(s) with the one move. This adds considerable depth to the parent game, and means that a hard upper limit is not necessarily a bad thing; play can get quite confusing for higher levels, despite the fact that the recursive metarule sits easily and transparently within the parent game's rules.

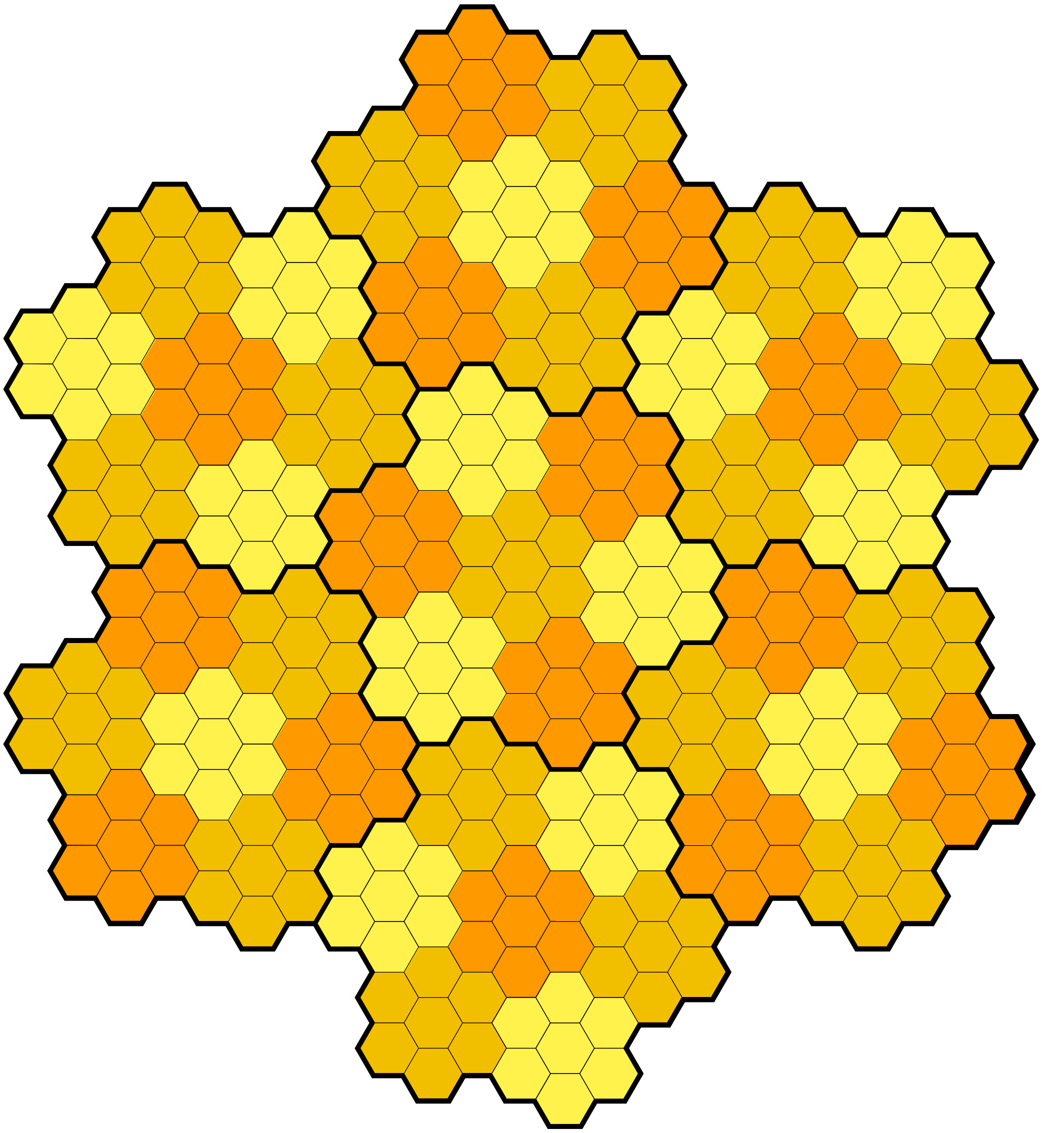

Fig. 6. A Gosper Island board for playing level 3 septant games.

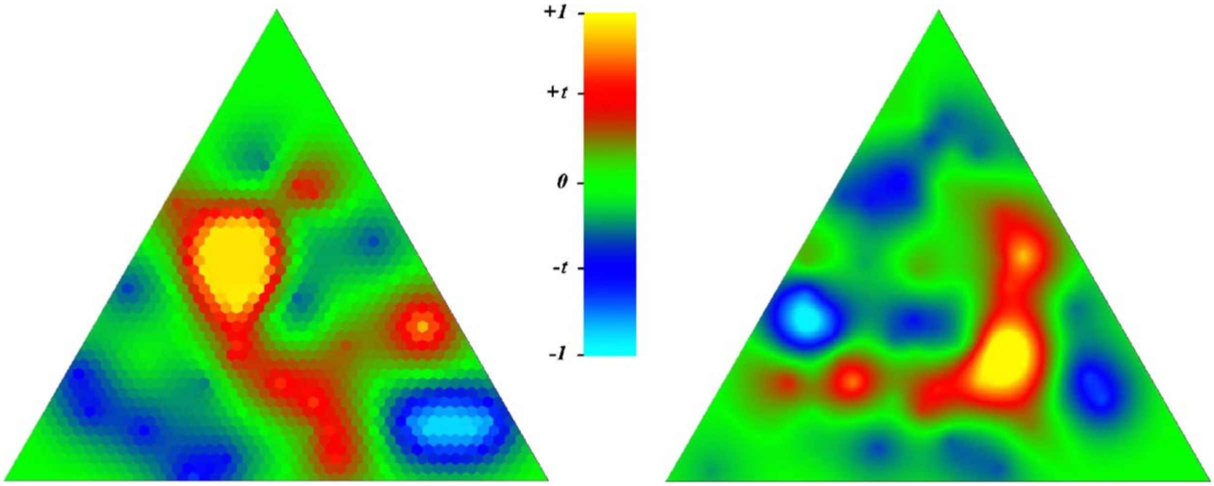

Fig. 7. Low and high-resolution games of Py in progress.

Fig. 5 constitutes a simple puzzle to give a taste of recursive play: Next move to win, regardless of whose turn it is. Note that Black, whose current score is 8, must win games on all three levels to win the overall match, while White, whose current score is 9, need only win on two levels.

There are several ways in which a given game may be subdivided. For instance, a game of size n may be divided into overlapping subgames of size $n-1$, $n-2$ etc, or may be divided into a greater number of non-overlapping subgames, such as a 12×12 game into nine 4×4 subgames. However, the best results were found to come from subdividing a game into the largest possible non-overlapping subgames at each level, making quarter-game subdivision the optimal solution in most cases.

Even though the hexagon is not a rep-tile, recursive games can be implemented on a hexagonally shaped board using a fractal known as the Gosper Island [4], as shown in Fig. 6. The resulting games are *septant* in nature rather than quadrant, as each component subdivision yields seven subcomponents. Thus $d = 7$ in Eq. (1) and the total number of component games is $1+7+49 = 57$ for this board. Note that three hues must be used to distinguish subregions due to the trivalent nature of the tiling.

There also exist abstract games played on boards that exhibit self-similar subdivision, such as the GRYB gaming system and Cliff Pickover's Fractal Fantasies [5]. However, these are distinct from recursive games as only the board is recursively defined, not the game itself.

4. Continuous games

Py (short for "Potential Y") is a connection game played according to the rules of Y, but with charges on a continuous potential field rather than with pieces on a discrete grid [6]. The Positive player places a positive point charge on the field each turn, and the Negative player places a negative point charge on the field each turn. Each point charge affects a circular area of diameter d (where d typically equals 10% of the length of a board edge) with linear falloff. Players are penalized for playing in areas of their opponent's polarity, to discourage copycat strategies. Rather than being an actual board game, Py is a theoretical game that requires a computer simulation of the potential field to be played.

The winner is the player who completes a field of their polarity, above a certain threshold, between the three sides of the board.

Fig. 7 shows two games of Py in progress. The low-resolution game on the left shows the underlying hexagonal nature of the playing field, while the game on the right shows the continuous nature of the game when played at a higher resolution.

The spectral chart between the two games describes the potential field's color coding. The thresholds for Positive and Negative are set to 0.5 (red) and -0.5 (blue), respectively, therefore Positive wins by connecting all three sides with a field that is red to yellow, and Negative wins by connecting all three sides with a field that is blue to cyan. Green boards areas are neutral.

Charges on the Py board slowly dissipate over time to return to neutral after n moves (typically 100 moves). This generally has little impact on the game, but adds a subtle element of dynamism which means that players must vigilantly monitor the entire board rather than take established connections for granted.

The fact that a period of 100 moves may have an impact on the outcome indicates that each game may consist of hundreds of moves. This sounds like a lot of moves for a turn-based game, but Py captures the intuitive essence of connection games without the need for precise piece placement; moves may be made quickly, and a few pixels' error will generally not have much effect. Py emphasizes high-level strategic planning to a greater extent than other connection games, and removes the trivial book-keeping of low-level combinatorial play to provide a purer form of connection game.

A Windows program for playing Py may be found at: http://members.optusnet.com.au/cyberite/py/py-1.htm.

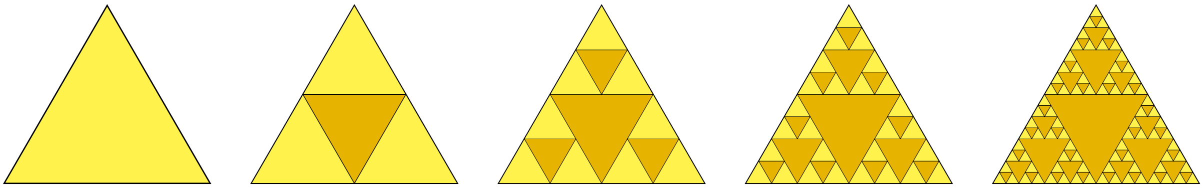

Fig. 8. A triangular board subdivided in the Sierpinski fashion.

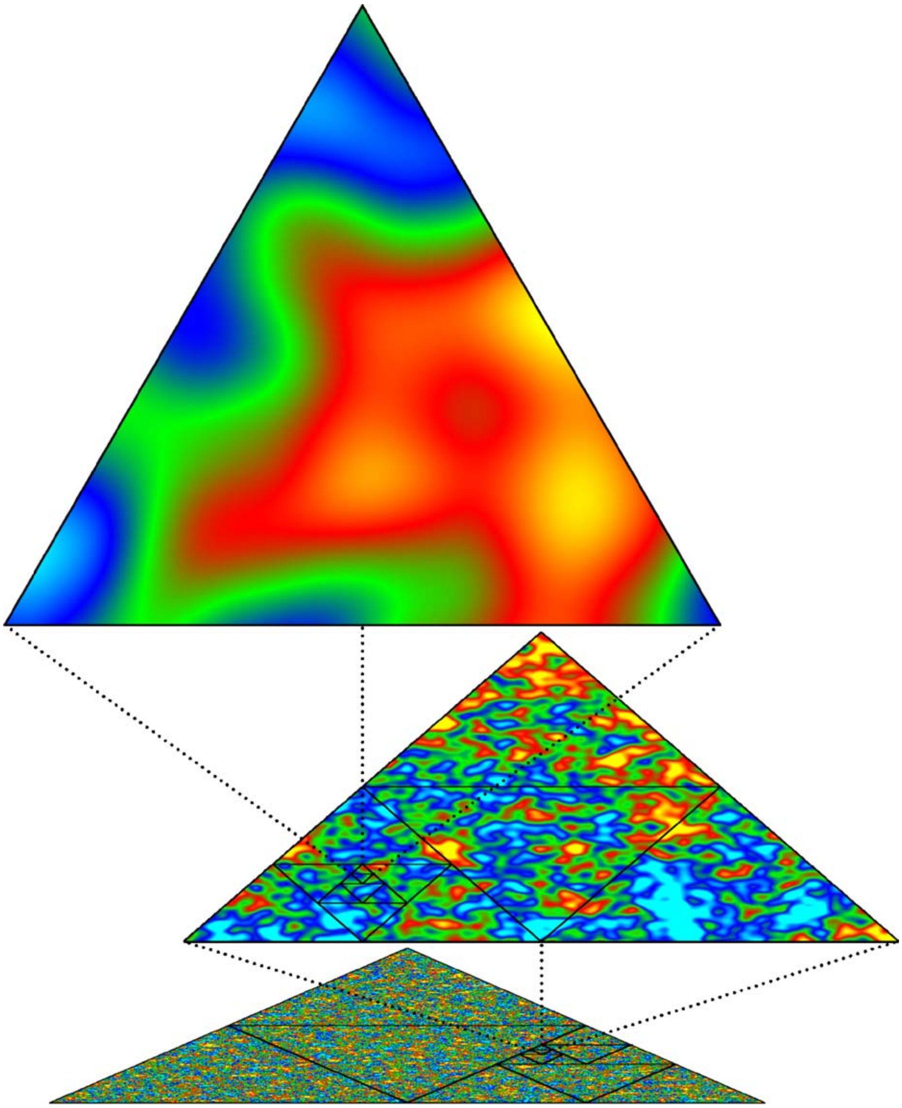

Fig. 9. A game of Fractal Py in progress with close-ups of subgames at levels 6 and 11.

This program is intended for tutorial purposes, and coordinates play against either a random computer player or another human opponent.

5. Fractal games

The well-known Sierpinski triangle subdivision, shown in Fig. 8, provides a convenient method for recursively subdividing a triangle to a given depth. Quadrant Y was based on the first step of this subdivision, highlighting the relationship between these recursive games and classic fractals (see Fig. 3). The fact that Py is continuous in nature removes the limitations of discrete board geometry, and allows the creation of a truly fractal game to an arbitrary depth. We will call this game Fractal Py.

Fig. 9 shows what a game of Fractal Py might look like. The bottommost triangle shows the supergame at level 1, the middle triangle shows a zoomed view of a portion of the game at level 6, and the topmost triangle shows a zoomed view of a portion of the game at level 11. Only those subdivisions relevant to the zoomed views have been explicitly shown, other subdivisions are implied. Note that this example is for illustrative purposes only and is not taken from a real game; the charge distribution is defined by Perlin noise [7] at different scales.

Fractal Py is played on the limit of subdivision dictated by the resolution of the input and display devices, as players must select a pixel at which to move each turn. It is not practical to play Fractal Py to an infinite depth; the score would have to be determined for an infinite number of component games, hence the potential field would have to be quantized at some point and a limit implied. In addition, each move in an infinite game would effectively be played on an infinite number of levels, making it impossible to develop sensible strategies. It is difficult enough planning moves for two or three recursive levels, as discussed above.

Fractal Py has not been implemented yet and remains a theoretical proposition. It is envisaged that it would provide a challenging game requiring a sensible recursion limit to remain comprehensible; the 11 levels shown in Fig. 9 may already be too many. In any case, the principle behind Fractal Py provides a technique for turning existing games into fractal ones.

6. Conclusion

This paper demonstrates how a recursive subdivision rule may be applied to some board games to give a new class of fractal and fractal-like games. This metarule tends to fit naturally within the parent game's rules and adds a challenging degree of complexity to a given game. It enhances the depth of many excellent games and could possibly inject life into some less interesting ones.

Future work might include tests to determine those games (not just connection games) for which the recursive metarule is suitable, including the optimal board size and level of subdivision in each case. In addition, it would be interesting to implement a Fractal Py player to gauge the success of this type of fractal game, and to see how it compares with its fractal-like counterparts.

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