GPA – a tool for fluid scalability analysis of massively parallel systems
QEST 2011

Anton Stefanek, Richard A. Hayden, Jeremy T. Bradley
{as1005,rh,jb}@doc.ic.ac.uk

Department of Computing, Imperial College London

September 7, 2011
Massively parallel Markov models

Client

Server
Massively parallel Markov models

Client
request

Client_waiting

Server
request

Server_get

Introduce a tool GPA that gives convenient access to these existing range of techniques to efficiently analyse metrics involving these populations.
Massively parallel Markov models

![Diagram of Massively parallel Markov models]

- Client
  - Client_waiting
  - Client_think

- Server
  - Server_get

request

data

request

data

Existing range of techniques to efficiently analyse metrics involving these populations

Introduce a tool GPA that gives convenient access to these...
Massively parallel Markov models

Client

Client_waiting

Client_think

Server

Server_get

think

request

data

Existing range of techniques to efficiently analyse metrics involving these populations

Introduce a tool GPA that gives convenient access to these

Chemical equations,
Massively parallel Markov models

Client

Client_waiting

Client_think

Server

Server_get

Server_broken

think

request

data

request

data

reset

break
Massively parallel Markov models

▶ \( N_C \) and \( N_S \) copies of identically behaved clients and servers
Massively parallel Markov models

- \( N_C \) and \( N_S \) copies of identically behaved clients and servers
- **CTMC states** \((C(t), C_w(t), C_t(t), S(t), S_g(t), S_b(t)) \in \mathbb{N}^6\)

Process algebra (PEPA)
Chemical equations, ...
Massively parallel Markov models

- $N_C$ and $N_S$ copies of identically behaved clients and servers
- CTMC states $(C(t), C_w(t), C_t(t), S(t), S_g(t), S_b(t)) \in \mathbb{N}^6$
- Existing range of techniques to efficiently analyse metrics involving these populations
Massively parallel Markov models

- $N_C$ and $N_S$ copies of identically behaved clients and servers
- CTMC states $(C(t), C_w(t), C_t(t), S(t), S_g(t), S_b(t)) \in \mathbb{N}^6$
- Existing range of techniques to efficiently analyse metrics involving these populations

Introduce a tool GPA that gives convenient access to these...
Fluid analysis of Markov models

- Traditionally only simulation possible

$\frac{d}{dt} E[C(t)] = \cdots$

$\frac{d}{dt} \text{Var}[C(t)] = \cdots$

$\frac{d}{dt} E[\int_0^t S(u) du] = \cdots$

$\frac{d}{dt} E[S(t)] = \cdots$

$\frac{d}{dt} E[C(t)S(t)] = \cdots$

$\frac{d}{dt} \text{Var}[\int_0^t S(u) du] = \cdots$

- Used to derive passage time distributions, rewards, ...
Fluid analysis of Markov models

- Traditionally only **simulation** possible
Fluid analysis of Markov models

- Traditionally only simulation possible

\[
\frac{d}{dt} \mathbb{E}[C(t)] = \cdots
\]

\[
\frac{d}{dt} \text{Var}[C(t)] = \cdots
\]

\[
\frac{d}{dt} \mathbb{E}[\int_0^t S(u) du] = \cdots
\]

\[
\frac{d}{dt} \mathbb{E}[S(t)] = \cdots
\]

\[
\frac{d}{dt} \mathbb{E}[C(t)S(t)] = \cdots
\]

\[
\frac{d}{dt} \text{Var}[\int_0^t S(u) du] = \cdots
\]
Fluid analysis of Markov models

- Traditionally only simulation possible
- Means are continuous
Fluid analysis of Markov models

- Traditionally only simulation possible
- Means, variances and other moments are continuous
Fluid analysis of Markov models

- Traditionally only simulation possible
- Means, variances and other moments are continuous
- **Fluid techniques** derive systems of ODEs approximating these

\[
\frac{d}{dt} \mathbb{E}[C(t)] = \cdots
\]

\[
\frac{d}{dt} \mathbb{E}[S(t)] = \cdots
\]
Fluid analysis of Markov models

- Traditionally only simulation possible
- Means, variances and other moments are continuous
- Fluid techniques derive systems of ODEs approximating these

\[ \frac{d}{dt} \mathbb{E}[C(t)] = \cdots \quad \frac{d}{dt} \text{Var}[C(t)] = \cdots \]

\[ \frac{d}{dt} \mathbb{E}[S(t)] = \cdots \quad \frac{d}{dt} \mathbb{E}[C(t)S(t)] = \cdots \]
Fluid analysis of Markov models

- Traditionally only simulation possible
- Means, variances and other moments are continuous
- Fluid techniques derive systems of ODEs approximating these

\[
\frac{d}{dt} \mathbb{E}[C(t)] = \cdots \quad \frac{d}{dt} \text{Var}[C(t)] = \cdots \quad \frac{d}{dt} \mathbb{E}\left[ \int_0^t S(u)du \right] = \cdots \\
\frac{d}{dt} \mathbb{E}[S(t)] = \cdots \quad \frac{d}{dt} \mathbb{E}[C(t)S(t)] = \cdots \quad \frac{d}{dt} \text{Var}\left[ \int_0^t S(u)du \right] = \cdots 
\]
Fluid analysis of Markov models

- Traditionally only simulation possible
- Means, variances and other moments are continuous
- Fluid techniques derive systems of ODEs approximating these

\[
\frac{d}{dt} \mathbb{E}[C(t)] = \cdots \quad \frac{d}{dt} \text{Var}[C(t)] = \cdots \quad \frac{d}{dt} \mathbb{E} \left[ \int_0^t S(u)du \right] = \cdots
\]

\[
\frac{d}{dt} \mathbb{E}[S(t)] = \cdots \quad \frac{d}{dt} \mathbb{E}[C(t)S(t)] = \cdots \quad \frac{d}{dt} \text{Var} \left[ \int_0^t S(u)du \right] = \cdots
\]

- Used to derive passage time distributions, rewards, \ldots
GPA – overview

Input file
GPA – overview

Input file

Parameters definition → Model description → Analyses description

CTMC → Analysis

Analysis

Sim. → ODEs
GPA – overview

- Parameters definition
- Model description
- Analyses description

Input file

CTMC

Analysis

ODEs

Implementation

Java
MATLAB
GPA – overview

- Parameters definition
- Model description
- Analyses description

Input file

- CTMC
- ODEs

Analysis

Implementation

- Parameters
- Moments

Passage time CDF Rewards

Optimisation

Moments
GPA – overview

Input file

Parameters

Model description

Analyses description

Analysis

CTMC

ODEs

Sim.

Implementation

Parameters

Moments

Passage time CDF

Rewards

4/9
GPA – overview

Parameters definition → Model description → Analyses description

Input file

CTMC → ODEs → Sim.

Parameters → Implementation

Moments → Passage time CDF Rewards
GPA – overview
Examples

![Graph showing probability of client finishing before time $t$.]

- Probability $P(t)$ is shown on the y-axis, ranging from 0 to 1.
- Time $t$ is shown on the x-axis, ranging from 0 to 40.
- The red line indicates the probability of the client finishing before time $t$.

![Graph showing mean reward $E[\text{reward}]$ over time $t$.]

- Mean reward $E[\text{reward}]$ is shown on the y-axis, ranging from 0 to 1,500.
- Time $t$ is shown on the x-axis, ranging from 0 to 40.
- The blue line represents the mean reward, and the dashed blue lines represent the 95% confidence interval $\pm 1.95\text{s.d.}$
Examples

SLA:
\[ P(t \leq 15.0) \geq 0.95 \]

Time, \( t \)
Probability
Client finished before \( t \)

Mean reward \( \mathbb{E}[\text{reward}] \pm 1.95\text{s.d.} \)

\( t \)
Mean reward
\( \mathbb{E}[\text{reward}] \pm 1.95\text{s.d.} \)

\( t \)

\( 0 \)
\( 500 \)
\( 1,000 \)
\( 1,500 \)

\( 0 \)
\( 500 \)
\( 1,000 \)
\( 1,500 \)
Example – parameter sweeping & optimisation
Example – parameter sweeping & optimisation
Example – parameter sweeping & optimisation
Example – parameter sweeping & optimisation

![Graph showing the relationship between reward, \( N_S \), and \( r_{\text{break}} \). The graph has a 3D surface plot with the y-axis representing reward, the x-axis representing \( N_S \), and the z-axis representing \( r_{\text{break}} \). The surface peaks at a maximum point, indicating the optimal values for \( N_S \) and \( r_{\text{break}} \) for maximizing reward.]
Example – different input models

Simple circadian clock model given as chemical equations:

\[ \mathbb{E}[R(t)] \]
\[ \mathbb{E}[C(t)] \]
Example – different input models

Simple circadian clock model given as chemical equations:

Simulation

ODEs

\[ E[R(t)] \]

\[ E[C(t)] \]
Example – different input models

Simple circadian clock model given as chemical equations:

Simulation

\[ E[R(t)] \]
\[ E[C(t)] \]

ODEs – 2nd order normal closure

\[ E[R(t)] \]
\[ E[C(t)] \]
Conclusion

- GPA – implementing fluid techniques for population CTMCs
- Supports models in GPEPA and chemical equations
- Provides API support for further extensions
- Implementation improvements – GPU, parallel
- More sophisticated optimisation – Taylor models
Thank you!