Using Fluid Queues to Model Energy Storage and Distribution

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Energy storage and distribution is an increasingly important problem as renewable (and generally unreliable) energy sources supply more of our power needs.

In this extended abstract we consider a simple network of energy suppliers, consumers and buffers modelled as fluid queues. Analytical evaluation of networks of fluid queues is hard as even tandem networks do no have a product form solution. We improve an approximation algorithm previously introduced to work in this new setting and evaluate our results using the Java JINQS simulator.

1 Introduction

Last summer the state of Texas had electricity generating capacity nearly twice that of demand, but the network operator was near imposing rolling blackouts as demand did not match supply [3]. Much of Texas' generating capacity is in renewables whose generating peaks do not coincide with the peaks in energy demand. As we move to generating more of our power from renewable sources, energy storage will become an important issue.

In this paper we introduce a network of fluid queues as a model of an energy generation and storage network. We show how an approximation algorithm [7, 9] can be used to compute response metrics of interest in this network, extending the algorithm using recent analytical results for the busy period of a fluid queue [10] so as to apply to the new situation encountered here.

This allows us to compute approximate moments of the busy period a consumer will experience in the network. This information would be useful when choosing network parameters within constraints available to designers of such a system.

In section 2 we introduce fluid queues and fluid queue networks and in section 3 we show how the approximation algorithm can be applied. We conclude with a discussion of future work.

2 Fluid queues and networks of fluid queues

2.1 Fluid queues

A fluid queue is a bivariate stochastic process (J_t, X_t) with an associated input rate vector λ and service rate scalar μ. The process J_t is a continuous time Markov chain on the states $\{1, 2, \ldots, n\}$ and $\mathbf{r} = \lambda - \mu \cdot 1$ is an n-dimensional vector. X_t is a stochastic process such that at time t, when $J_t = i$,

$$\frac{\mathrm{d}X_t}{\mathrm{d}t} = \begin{cases} r_i & \text{if } X_t > 0 \\ \max(r_i, 0) & \text{if } X_t = 0. \end{cases}$$

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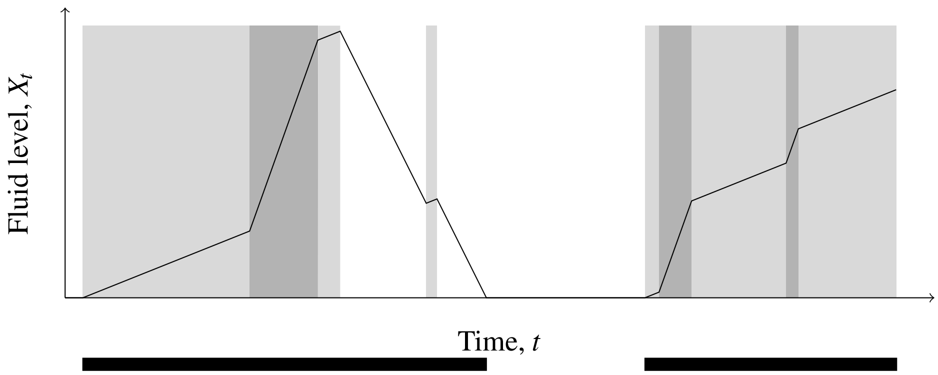

Figure 1: Sample trace from the fluid level at node 1. The areas with a light grey highlight correspond to the times when one source was feeding the node and the areas with a dary grey highlight correspond to the times when both sources were feeding the node. The busy periods are highlighted with a thick black line. The first is a complete busy period, the second has not finished in the duration of this trace.

Fluid queues are a sub-class of piecewise deterministic Markov processes. A sample trace from a fluid queue with two inputs (node 1 in Figure 2) is shown in Figure 1. The output process of a fluid queue Y_t is given by

$$Y_t = \begin{cases} \mu & \text{if } X_t > 0 \\ \lambda_i & \text{if } X_t = 0. \end{cases}$$

Such models have been studied extensively in the literature. See [11] for stationary distribution results and [10] for computation of the busy period distribution. The busy period is the time period for which the fluid queue contains fluid (has positive fluid level). A busy period is highlighted with a bold line in Figure 1.

2.2 Fluid queue networks

In a fluid queue networks we consider a number of fluid queues linked together in a feedforward manner. We define a routing matrix P with rows i that describe the proportion of fluid leaving node i that is sent to nodes $1, 2, \ldots, n$. In the network considered here the routing matrix is particularly simple as all fluid leaving a source or node is routed to only one place. More generally we can apply the ideas here to networks where nodes can be numbered in such a way that P is an upper diagonal matrix with zeros on the diagonal. We do not consider networks with loops/cycles.

3 Example

3.1 Model description

We consider the example network in Figure 2. There are four independent on/off sources, each of which produces power for 1/3 of the time on average. A 33% capacity factor is a reasonable approximation to that of a wind turbine (typically 20–40% [2]).

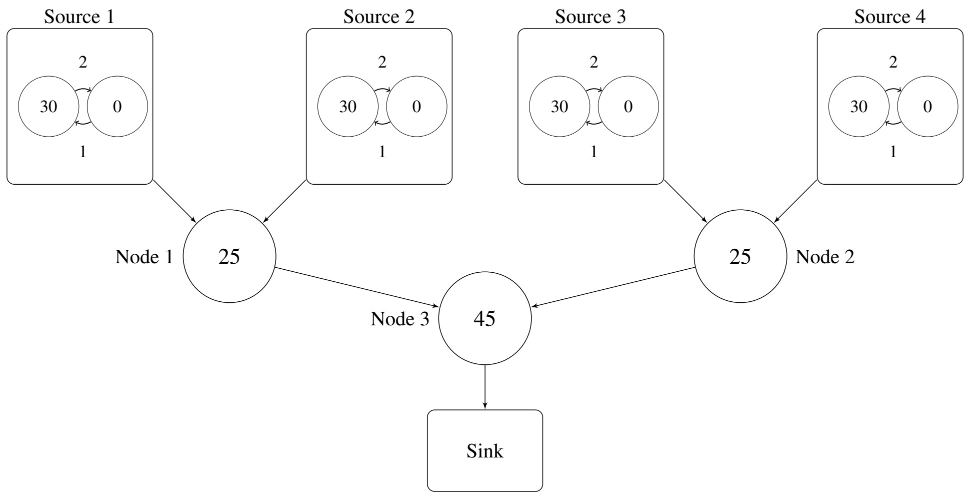

Figure 2: The example network considered in this paper. Fluid is generated at each of the four sources at rate 30.

In Figure 2, nodes 1 and 2 represent storage facilities that smooth out the intermittent power produced by the four sources. Node 3 represents a customer who requires power at rate 45. The four sources each have an on rate of 30.

We compute an approximation for the moments of the busy period at node 3. The first moment (the mean busy period) corresponds to the average length of time for which the customer at node 3 has their power demands satisfied.

3.2 Approximation algorithm

We start the algorithm with details of the output of the source nodes. We work through each node in the tree-like network approximating the output as a Markov modulated flow, using these approximations as inputs to approximate the behaviour at sebsequent nodes.

We start at nodes 1 and 2 using the method in Field and Harrison [9]. They note that the holding time of the off period in the output process from these nodes has the same distribution as the off period of the combined input process–exponentially distributed with parameter 2. The on period of the output from nodes 1 and 2 is approximated by an exponential distribution with parameter chosen so as to conserve throughput.

The output of nodes 1 and 2 is therefore approximated by $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ 2 & -2 \end{pmatrix}$ on the state space $\{25, 0\}$.

Node 3 has two inputs, each approximated by the above matrix and rates and we are interested in the busy period at this node. We use the recent Field and Harrison result [8] to compute the busy period at this node.

For the 3 state approximation we choose parameters as in [9] using least squares to match second moments of the busy period. Using the JINQS simulation tool [8] we compute the actual values and see the approximation performs well.

	2 state approximation	3 state approximation	Simulation value
Mean busy period at node 3	2.22	2.26	2.26

(Due to complications in dealing with repeated eigenvalues, the approximations actually assumed the service rates were 24.9 and 25.1 at nodes 1 and 2.)

4 Future work

In this paper we have applied a recent theoretical result of Field and Harrison to improve the applicability of an approximation algorithm previously proposed by the same authors. We have considered a simple example and seen good performance.

Much work remains to improve the approximation algorithm and create a useful tool. The approximation proposed here does not deal with networks that have *small flows*. These occur when the buffer is empty and fluid is arriving at a rate less than μ. The fluid passes straight through the buffer and is immediately an input to the next node.

Also, the network considered here was particularly simple as there were no dependencies between flows. Suppose we had taken sources 2 and 3 and combined them in to one on/off source of rate 60, the output of which was split between nodes 1 and 2 equally. This would increase the mean busy period to 3.00, while the approximation (ignoring this dependence) would remain unchanged.

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