Adaptive Joint Detection and Estimation in MIMO systems: A Hybrid Systems Approach

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The material in this paper was presented in part at the Symposium on Signal Processing for Communications, IEEE GLOBECOM conference, San Francisco, CA, December 2003. H. Kulatunga* and V. Kadirkamanathan are with the Dept. of Automatic Control and Systems Engineering, University of Sheffield, Mappin Street, Sheffield, S1 3JD, UK (Tel: +44 114 222 5250, Fax: +44 114 222 5661 email: {cop01thk*, visakan}@sheffield.ac.uk).

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Abstract

An adaptive receiver based on hybrid system theory is developed for a multiuser multiple-input multiple-output (MIMO) fading CDMA system. The basic idea is to treat the transmitted symbols and channel gains as unknown states (discrete and continuous) within a hybrid systems framework. The Bayesian inference based state estimation is derived using multiple model theory resulting in an optimal joint sequence estimator which is shown to be intractable in its computational complexity. A sub-optimal receiver (IMM-SIC) is then derived based on the well-known Interacting Multiple Model (IMM) algorithm and successive interference cancellation (SIC) scheme. The paper shows the specific approximations made to the probability densities of the optimal receiver in deriving IMM-SIC receiver with complexity linear in number of users. This receiver design is well suited for online recursive processing of space-time coded (STC) CDMA system where the decoding stage is incorporated within the multiple model framework.

Index Terms

Joint estimation and detection, multiple models, multiuser detection (MUD).

EDICS Category: 3-COMM

I. Introduction

A time-selective MIMO wireless system exhibits temporal diversity and spatial diversity. This leads to improved bandwidth utilization at the expense of increased computational complexity in such a system. STC can be employed to further exploit temporal and spatial diversity. ST block coding (STBC) was first introduced by [1] for an unmodulated two transmitter antennae case with known channel information and flat fading. This was later generalized for N_T transmitters [2]. STBC significantly improves detection and increases system capacity. Recent literature [3], [4], [5], [6] featuring the use of diversity techniques rely on the underlying structure induced by spreading waveforms of DS-CDMA systems and/or the signal structure induced by antenna array for interference suppression.

A considerable amount of research has been devoted to signal detection in flat Rayleigh fading channels. Suboptimal algorithms proposed involve two-stage receiver structures or joint channel estimation and symbol detection. The former advocates channel estimation implemented by a linear predictor and signal detection facilitated by per-survivor processing [7], decision feedback [8], pilot symbols [9], [10] or combination of all the above [11]. Other suboptimal solutions include hidden Markov models (HMM), Kalman filtering [12], H^∞ filtering [13], [14] and methods based on the expectation-maximization (EM) algorithm [15]. Furthermore, joint estimation and detection techniques are developed based on iterative processing [16] or sequential Monte Carlo filtering based methods such as Gibbs sampling [17] or mixture Kalman filtering (MKF) [18].

Motivation for our approach is multiple model filtering, popular in target tracking, due to superior performance when the system states and noise levels are random; since the wireless environment has high levels of uncertainty and interference we have investigated the extension of a multiple model based method for joint estimation and detection problem in wireless communication. Multiple model approach uses a set of models that differ in their association hypothesis and in their driving and measurement noise levels. A filter is set up for each model and the probability that each one of these model associated hypothesis gave rise to a certain measurement is obtained. The set of associated hypothesis adapts with time based on the latest set of measurements. A popular algorithm is the Interacting Multiple Model (IMM) method proposed by [19], [20] which was used for symbol detection and channel equalization in static single-user ISI channels [21]. In this paper we have introduced the MIMO multiuser communication signal model within a hybrid system framework and used models based on associated hypothesis to solve the joint estimation and detection problem in Rayleigh flat fading channels and derived the IMM-SIC algorithm (based on the IMM principle) to estimate the model states.

The proposed method is in contrast with other Bayesian receivers. Monte Carlo filtering (Gibbs sampling) based methods, for example [22],[23], where the receiver exhibits massive parallelism due to Gibbs multiuser detector. Other publications with similar approach and massive parallelism include [24], a recursive MKF algorithm which also approximates the channel distribution with a mixture of Gaussians. MKF is similar to Rao-Blackwellised Particle Filtering (RBPF) [25] and [26] and has been implemented for the communication signal model in [24], [27], but is limited to a single-user flat-fading channel, and in [28] for a MIMO single-user channel with an iterative detector. Joint maximum a posteriori (MAP) estimation and detection has also been approached in [29], [30] which are trellis based algorithms with complexity exponential in time unless pruning [29] (Reduced-state Sequence Estimation (RSSE)) or decision feedback [30] is employed. Similarly [31], [32] propose a forward-backward recursion for a trellis based joint MAP detector where [32] expands on the work of [31] to include any arbitrary modulation scheme with a fixed trellis size and is conceptually similar to the Per-survivor Processing (PSP), and the number of trellis states is in the order of channel memory and symbol block size. This is in contrast with our online joint detection algorithm where complexity of the state-space does not grow with channel memory or sequence length. Considering an uplink, mobile station (MS) to base station (BS), consideration needs to be given to multiple users' multi-paths which vary independently with time. There are a number of publications that consider uplink CDMA. [33] considers slow fading channels and presents an iterative procedure using an approximate sequential EM (AEM) receiver for quasi-static channel estimation and detection which has a complexity exponential in number of users per bit and requires decision feedback initialization for each EM iteration. In [34] another iterative technique combining the Gibbs sampler with a group-blind decorrelator is proposed for an uplink CDMA system.

The paper is organised as follows. In Section II, the system model of a MIMO system is presented. Section III introduces the hybrid system framework for a MIMO system and Section IV the multiple model based estimation of this framework. Section V develops the suboptimal receiver for a general MIMO system and finally extends it to the special case of STC MIMO and introduces a ST decoding strategy based on the Markov property of the discrete hybrid states (symbols). Section VI presents simulation results and Section VII concludes our presentation.

II. System Description

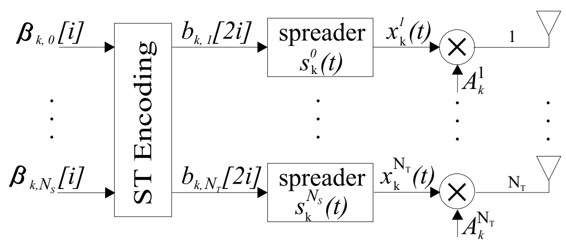

Fig. 1. Transmitter design for user k.

Consider an uplink MIMO system with K active users and N_T transmitter (Tx) antennae on each user terminal and N_R dimensional receiver (Rx) antenna array at the base station. The block diagram of the transmitter end of such a system is shown in Fig. 1. In discussions to follow, the number of independent signal streams per user (N_s+1) is equal to the number of transmitters per user (N_T), i.e. $N_T = N_s+1$. A number of typical assumptions are made and the system description is developed.

Assumption 2.1: Transmission symbol sequence $\{\beta_{k,n_s}[i]\}$ is generated by a Markov source. The spread-spectrum signal can be considered as a sequence from a finite state Markov chain.

Remark: General source coding of messages, synchronous multiuser transmission and sampling will lead to a correlated sequence of symbols such as those from a Markov source. The independent identically distributed (i.i.d.) sequence is a special case and will be considered in Section VI. Assuredly this receiver can be extended to an asynchronous CDMA channel.

For the n_sth i.i.d. BPSK transmission stream the transition probability matrix $[\Psi_{n_s}]_{S\times S}$ is known with all of its elements being 0.5, for example the conditional probability $P(\beta_{k,n_s}[i] = +1|\beta_{k,n_s}[i-1] = -1) = [\Psi_{n_s}]_{21} = 0.5$ where $[\cdot]_{21}$ is the $(2,1)$th matrix element. With $n_s = 0, 1$ ($N_T = 2$) the combined transition matrix is the Kronecker product of the individual transmission stream probability matrices; $\Pi_0 = [\Psi_1 \otimes \Psi_0]_{\mathcal{Q}\times\mathcal{Q}}$ where $\mathcal{Q} = S^2$ and $\otimes$ denotes Kronecker product operation. This can be generalised to $n_s = 0, \ldots, N_s$ and the transition matrix will be $\Pi_0 = [\Psi_{N_s} \ldots \otimes \Psi_2 \otimes \Psi_1 \otimes \Psi_0]_{\mathcal{Q}\times\mathcal{Q}}$ where $\mathcal{Q} = S^{N_T}$.

Assumption 2.2: The DS/CDMA MIMO system is Alamouti Space-time (ST) coded at symbol level ([1], [2]) unlike ST coding performed at block level ([35], [36]) for two transmitters.

Remark: This can be generalised to $N_T > 2$ by resorting to generalised complex orthogonal design (GCOD) [2] with $N_T = N_s + 1$, which is analogous to Alamouti STC but for more than two transmitters. For clarity we limit our discussion to $N_T = 2$ transmit antennae. Alamouti ST coding generates the following 2 x 2 transmission matrix where symbol at time-instant $2i + 1$ from one antenna is the time reversed conjugate of symbol at time-instant $2i$ from other antenna,

$$\begin{pmatrix} b_{k,1}[2i] & b_{k,2}[2i] \\ b_{k,1}[2i+1] & b_{k,2}[2i+1] \end{pmatrix} = \begin{pmatrix} \beta_{k,0}[i] & \beta_{k,1}[i] \\ -\beta^*_{k,1}[i] & \beta^*_{k,0}[i] \end{pmatrix} \quad (1)$$

where $n_s = 0, 1$ and $b_{k,n_t}[i]$ denote the transmitted signal from Tx n_t where $n_t = 1, 2$ for user k at time iT, with T being the symbol duration.

Π_1 is defined as the transition probability matrix for Alamouti STC transmission for a BPSK modulation, and is given in Table I. Elements of the matrix Π_1 are either 0 or 1. For example, the first row in Table I contains the elements $0, 1, 0, 0$ indicating the certainty in the symbol transmission following encoding, i.e. if the symbol pair transmitted at time $2i$ is $[-1, +1]$, then the symbol pair transmitted at time $2i + 1$ is with certainty $[-1, -1]$. Π_1 can be generalized to any orthogonal transmission design [2] for $N_T > 2$ MIMO systems. The consequence of Alamouti STC is that the combined transition probability matrix of the transmission stream will alternate between the matrix at $2i+1$ being Π_1 (Assumption 2.2) and at time $2i$ being Π_0 (Assumption 2.1) for $i = 1, 2, \ldots, M$ where M is the sequence length. For BPSK modulation the size of the signal constellation is $S = 2$, the extension to the general case is straightforward.

In DS/CDMA each n_sth symbol sequence is assigned an orthogonal spreading sequence.

$$s_k^{n_s}(t) = \sum_{n=1}^{N} s_{k,n}^{n_s} p_{T_c}(t - (n-1)T_c) \tag{2}$$

The normalized modulation waveforms $s_k^{n_s}(t)$ are zero for $t \notin [0, T]$. $s_{k,1}^{n_s} s_{k,2}^{n_s} \ldots s_{k,N}^{n_s}$ is a N length signature sequence of $+1$s and -1s assigned to kth user, and p_{T_c} is a unit-amplitude pulse of duration T_c (where $NT_c = T$). Each of the KN_T transmission signals, are written as,

$$x_k^{n_t}(t) = \sum_{i=1}^{M} A_k^{n_t} b_{k,n_t}[i] s_k^{n_s}(t - iT) \quad \text{for } n_t = 1, \ldots, N_T, k = 1, \ldots, K. \tag{3}$$

For clarity of exposition, the rest of Section I and Section II - IVB are developed for the general MIMO case that encompasses Alamouti ST encoding as a special case due to Assumption 2.1. The specific effect of ST encoding is considered in Section VC when application of IMM-SIC for Alamouti STC is considered.

1) Channel Model: Signal $x_k^{n_t}(t)$ is passed through a channel which can cause amplitude and phase shifts. In this work the channel is modeled in Cartesian co-ordinate form ($c = c^R + \mathbf{j}c^I$) in the complex plane. The sequence $\{c_{kl}^{(n_t,n_r)}\}$ is the channel gain between n_tth Tx and n_rth Rx of k-th user's lth multipath which includes effects of the transmitter and receiver as well as the amplitude and phase of the channel response.

Assumption 2.3: The phase and amplitude of the composite channel gain $c_{k,l}^{(n_t,n_r)}$ is Rayleigh distributed and constant over the symbol duration T, i.e. $c_{k,l}^{(n_t,n_r)}(t) = c_{k,l}^{(n_t,n_r)}[i]$ for $iT \leq t < (i+1)T$.

2) Observation Model: If $v^{n_r}(t)$ is the complex additive white Gaussian noise, the baseband equivalent signal at receiver n_r [6] is,

$$r^{n_r}(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} [c_{k,l}^{(1,n_r)}[t] x_k^1(t - \tau_{k,1} - \Upsilon_{k,l}) + c_{k,l}^{(2,n_r)}[t] x_k^2(t - \tau_{k,2} - \Upsilon_{k,l})] + v^{n_r}(t) \tag{4}$$

1	$\{ b_{k,1}[2i], b_{k,2}[2i] \}$			
$\{ b_{k,1}[2i+1], b_{k,2}[2i+1] \}$	[-1, -1]	[-1,+1]	[+1, -1]	[+1,+1]
[-1, -1]	0	1	0	0
[-1,+1]	0	0	0	1
[+1, -1]	1	0	0	0
[+1,+1]	0	0	1	0

TABLE I

Transition Probabilities for Alamouti STC MIMO system

Assumption 2.4: The system is synchronous so that channel dispersion i.e. $\tau_{k,n_t} = 0$. Furthermore, there are L multipaths between each user's Tx n_t and the Rx n_r with delay for l-th path represented as $\Upsilon_{k,l}$ and $\Upsilon_{k,l} = \frac{l-1}{W}$, $W = \frac{1}{T_c}$.

Denote $x_{kl}^{n_t}(t) = x_k^{n_t}(t - \Upsilon_{k,l})$, and the resulting continuous time signal $r^{n_r}(t)$ is given by,

$$r^{n_r}(t) = \sum_{k=1}^{K}\sum_{l=1}^{L}[c_{k,l}^{(1,n_r)}[t]x_{kl}^1(t) + c_{k,l}^{(2,n_r)}[t]x_{kl}^2(t)] + v^{n_r}(t) \tag{5}$$

Remark: The synchronous case of model (5) can be considered equivalent to the synchronous multiuser system with (KN_TL) users where it is sufficient to consider the received signal at ith symbol interval. In the receiver model the unique signature sequence assigned to each Tx of each user is shifted by $(l-1)$ chip durations to represent each multipath and $T \gg \frac{L}{W}$ therefore ISI due to channel dispersion can be neglected [37].

Sampling the received signal (5) at chip rate $\frac{1}{T_c}$, and where the channel is constant over a symbol duration NT_c, results in the discrete-time representation below,

$$r^{n_r}[iN+n] = \frac{1}{\sqrt{N}}\sum_{k=1}^{K}\sum_{l=1}^{L}[c_{kl}^{(1,n_r)}[i]\underbrace{A_k^1 b_{k,1}[i]s_{k,n-l+1}^0}_{x_{kl}^1[n']} + c_{kl}^{(2,n_r)}[i]\underbrace{A_k^2 b_{k,2}[i]s_{k,n-l+1}^1}_{x_{kl}^2[n']}] + v^{n_r}[n] \tag{6}$$

Assumption 2.5: $\{v^{n_r}[n]\}_{n_r=1}^{N_R}$ is a Gaussian random complex variable with zero-mean and variance ρ_v^2,

$$v^{n_r}[n] \sim \mathcal{N}(0, \rho_v^2) \tag{7}$$

The chip rate sampled signal sequence $\{r^{n_r}[iN+1], \ldots, r^{n_r}[(i+1)N]\}$ after serial to parallel conversion is,

$$\mathbf{r}^{n_r}[i] = \frac{1}{\sqrt{N}}\sum_{k=1}^{K}\sum_{l=1}^{L} c_{kl}^{(1,n_r)}[i]\underbrace{A_k^1 b_{k,1}[i]\mathbf{s}_{kl}^0}_{\mathbf{x}_{kl}^1[i]} + c_{kl}^{(2,n_r)}[i]\underbrace{A_k^2 b_{k,2}[i]\mathbf{s}_{kl}^1}_{\mathbf{x}_{kl}^2[i]} + \mathbf{v}^{n_r}[i] \tag{8}$$

$$\begin{aligned} \text{where,} \quad \mathbf{x}_{kl}^{n_t}[i] &= A_k^{n_t} b_{k,n_t}[i]\mathbf{s}_{kl}^{n_s} \\ \mathbf{s}_{kl}^{n_s} &= \begin{cases} [s_{k,N-l+2}^{n_s}, \ldots s_{k,N}^{n_s}, s_{k,1}^{n_s} \ldots s_{k,N-l+1}^{n_s}]^\top & \text{for } l > 1, \\ [s_{k,1}^{n_s}, \ldots s_{k,N}^{n_s}]^\top & \text{for } l = 1, \end{cases} \end{aligned} \tag{9}$$

The following matrix manipulations lead to the MIMO signal model at the receiver. First we stack the N_R sampled signals which results in,

$$\mathbf{r}[i] = \frac{1}{\sqrt{N}}\sum_{k=1}^{K}\sum_{n_t=1}^{N_T}\left[\ \mathbf{c}_{k,1}^{n_t}[i] \otimes \mathbf{x}_{k1}^{n_t}[i] + \quad \ldots \quad + \mathbf{c}_{k,L}^{n_t}[i] \otimes \mathbf{x}_{kL}^{n_t}[i]\ \right] + \mathbf{v}[i] \tag{10}$$

where $\mathbf{r}[i]$ is of dimension $N_RN \times 1$ and $\mathbf{c}_{k,l}^{n_t}[i] = [c_{k,l}^{(n_t,1)}[i]^H \ldots c_{k,l}^{(n_t,N_R)}[i]^H]^H$ and $\mathbf{x}_{kl}^{n_t}[i] = [x_{kl}^{n_t}[iN]^\top \ldots x_{kl}^{n_t}[(i+1)N-1]^\top]^\top$. Noise vector $\mathbf{v}[i]$ is zero mean i.i.d. random vector independent of $\{\mathbf{c}_{k,l}^{n_t}\}_{k=1,n_t=1}^{K,N_T}$, $\{\mathbf{x}_{kl}^{n_t}\}_{k=1,n_t=1}^{K,N_T}$. The spatial separation between Rxs are typically proportional to the wavelength of the signal, resulting in no correlation between the received signals [38], [37], [39]. The R.H.S. of (10) is re-arranged for each user's multipath signals to have a per-Rx ordering,

$$\mathbf{r}[i] = \frac{1}{\sqrt{N}} \sum_{k=1}^{K} \sum_{n_t=1}^{N_T} \begin{bmatrix} [\mathbf{x}_{k1}^{n_t}[i] \ldots \mathbf{x}_{kL}^{n_t}[i]] & \ldots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \ldots & [\mathbf{x}_{k1}^{n_t}[i] \ldots \mathbf{x}_{kL}^{n_t}[i]] \end{bmatrix} \begin{bmatrix} \mathbf{c}_k^{(n_t,1)}[i] \\ \vdots \\ \mathbf{c}_k^{(n_t,N_R)}[i] \end{bmatrix} + \mathbf{v}[i] \quad (11)$$

where $\mathbf{c}_k^{(n_t,n_r)}[i] = [c_{k,1}^{(n_t,n_r)}[i]^H \ldots c_{k,L}^{(n_t,n_r)}[i]^H]^H$ and $\mathbf{I}_{N_R}$ is an identity matrix of size N_R. Substituting (9) in (11),

$$\mathbf{r}[i] = \sum_{k=1}^{K} \sum_{n_t=1}^{N_T} \underbrace{\frac{1}{\sqrt{N}}(\mathbf{I}_{N_R} \otimes [\mathbf{s}_{k1}^{n_s}[i] \ldots \mathbf{s}_{kL}^{n_s}[i]])}_{\mathbf{S}_k^{n_s}}(A_k^{n_t} b_{k,n_t}[i]\mathbf{I}_{LN_R})\mathbf{c}_k^{n_t}[i] + \mathbf{v}[i] \quad (12)$$

where $\mathbf{c}_k^{n_t}[i] = [\mathbf{c}_k^{(n_t,1)}[i]^H \ldots \mathbf{c}_k^{(n_t,N_R)}[i]^H]^H$ and $\mathbf{S}_k^{n_s}$ is of dimension $N_R N \times LN_R$.

$$\mathbf{r}[i] = \sum_{k=1}^{K} \sum_{n_t=1}^{N_T} \mathbf{S}_k^{n_s}(A_k^{n_t} b_{k,n_t}[i]\mathbf{I}_{LN_R})\mathbf{c}_k^{n_t}[i] + \mathbf{v}[i] \quad (13)$$

which forms a received signal vector with a per-Rx ordering for multipath signals of KN_T transmission streams. (13) in matrix form is,

$$\mathbf{r}[i] = \sum_{k=1}^{K} \mathbf{S}_k \mathbf{A}_k \mathbf{B}_k[i]\mathbf{c}_k[i] + \mathbf{v}[i] \quad (14)$$

where $\mathbf{B}_k[i] = \mathrm{diag}(b_{k,1}\mathbf{I}_{LN_R}, \ldots, b_{k,N_T}\mathbf{I}_{LN_R})$, $\mathbf{A}_k = \mathrm{diag}(A_k^1\mathbf{I}_{LN_R}, \ldots, A_k^{N_T}\mathbf{I}_{LN_R})$, $\mathbf{S}_k = [\mathbf{S}_k^0 \ldots \mathbf{S}_k^{N_s-1}]$ and $\mathbf{c}_k[i] = [\mathbf{c}_k^1[i]^H \ldots \mathbf{c}_k^{N_T}[i]^H]^H$. Notation $\mathrm{diag}(\mathbf{X_1}, \mathbf{X_2})$ is a block diagonal matrix with matrices $\mathbf{X_1}$ and $\mathbf{X_2}$ on the diagonal. The composite received signal from all the receivers (14) is matched filtered to yield a KLN_RN_T vector $\mathbf{y}[i]$, whose $(k-1)LN_RN_T + (n_t-1)LN_R + (n_r-1)L + l$th component is the output of a filter matched to $\mathbf{s}_{kl}^{n_s}$. Thus kth user's matched filtered output is,

$$\mathbf{y}_k[i] = \begin{bmatrix} \mathbf{S}_k^0 & \ldots & \mathbf{S}_k^{N_s-1} \end{bmatrix}^\top \mathbf{r}[i] \quad (15)$$

where $\mathbf{y}_k[i] = [\mathbf{y}_k^1[i]^H, \ldots, \mathbf{y}_k^{N_T}[i]^H]^H$. Substituting (14) in (15) gives,

$$\mathbf{y}_k[i] = \sum_{k'=1}^{K} \mathbf{R}_{k,k'}\mathbf{A}_{k'}\mathbf{B}_{k'}[i]\mathbf{c}_{k'}[i] + \tilde{\mathbf{v}}_k[i] = \mathbf{R}_k\mathbf{A}\mathbf{B}[i]\mathbf{c}[i] + \tilde{\mathbf{v}}_k[i]$$

where $\mathbf{R}_k$ ($\mathbf{R}_k = [\mathbf{R}_{k,1} \ldots \mathbf{R}_{k,K}]$) is the correlation matrix of dimension $LN_RN_T \times KLN_RN_T$ for the kth user. The square submatrix denoted as $\mathbf{R}_{k,k'}$ is the correlation matrix between users k and k'. $[\mathbf{R}_{k,k'}]_{(n_t-1)LN_R+(n_r-1)N_R+l,(n_t'-1)LN_R+(n_r'-1)N_R+l'} = 0$ for $n_r' \neq n_r$ due to spatial separation at the receivers. $n_t, n_r, l/n_t', n_r', l'$ are the Tx,Rx, multipath indices for users k and k' respectively. Stacking the matched filtered outputs in the order of the users leads to,

$$\mathbf{y}[i] = \mathbf{R}\mathbf{A}\mathbf{B}[i]\mathbf{c}[i] + \tilde{\mathbf{v}}[i] \quad (16)$$

where $\mathbf{R}$ is a square matrix of dimension $KLN_RN_T \times KLN_RN_T$. Upper triangular Cholesky factorization [40] of the correlation matrix in (16) leads to $\mathbf{R} = \mathbf{U}\mathbf{U}^\top$, where $\mathbf{U}$ is a upper triangular matrix and $\mathbf{L} = \mathbf{U}^\top$. Sending the matched filtered signal through a filter $\mathbf{U}^{-1}$ yields a signal, $\mathbf{z}[i] = \mathbf{U}^{-1}\mathbf{y}[i]$ with white Gaussian observation noise $\bar{\mathbf{v}}[i]$,

$$\mathbf{z}[i] = \mathbf{LAB}[i]\mathbf{c}[i] + \bar{\mathbf{v}}[i] \tag{17}$$

with $\mathbf{z}_k^{(n_t,n_r)} = [z_{k1}^{(n_t,n_r)^H} \ldots z_{kL}^{(n_t,n_r)^H}]^H$, $\mathbf{z}_k = [\ldots, \mathbf{z}_k^{(n_t,n_r)^H}, \ldots]^H$ and $\mathbf{z} = [\mathbf{z}_1^H, \ldots, \mathbf{z}_K^H]^H$. $\mathbf{z}[i]$ is a sufficient statistic for determining channel states and symbols transmitted.

III. Hybrid System approach to the MIMO model

The observation model (17) relates the measurements (matched filtered noise whitened output) to unknown gains and symbols. The problem addressed in this paper is the joint estimation and detection of the unknown channel gain and transmit sequence in the presence of multiuser, multipath interference.

A. State Space System Model

Taking into account that the actual channel model is not known, the time-varying channel is tracked by a simple state-space model,

$$\mathbf{c}[i+1] = \mathbf{F}\mathbf{c}[i] + \mathbf{w}[i] \tag{18}$$

for some known $\mathbf{F}$,(typically with $\mathbf{F} = f\mathbf{I}_{KLN_RN_T}$ for some scalar $0 \ll f < 1$ [12]). Note that $f = 1$ will result in a Random-Walk model. $\mathbf{w}(i)$ is driving disturbance which is statistically independent and Gaussian distributed i.e., $\mathcal{N}(0, \rho_w^2\mathbf{I}_{KLN_RN_T})$. The corresponding state-space measurement equation will be,

$$\mathbf{z}[i] = \mathbf{LAB}[i]\mathbf{c}[i] + \bar{\mathbf{v}}[i] \tag{19}$$

Unfortunately, in many practical communication systems, we do not know the impulse response nor the channel order not to mention the channel gains varying over time. Therefore the objective is to find the transmitted symbol sequence from measurement sequence when channel coefficients and channel order are not known. To achieve this the MIMO system state-space model, (18),(19), is posed as a hybrid system and through state estimation we track the system's behavior along both its continuous state (channel gain) changes and its discrete state (symbol) changes. It is common to refer to the discrete state of the hybrid system as system's mode.

B. The proposed hybrid system framework

Consider the kth user in a BPSK MIMO system with $N_T = 2$ transmitters. $[b_{k,1}, b_{k,2}]$ is the symbol vector transmitted by user k at time-instant i, and can take one of four possible values as shown below.

$$\mathbf{b}_k = [b_{k,1}, b_{k,2}] \in \{[-1,-1],[-1,+1],[+1,-1],[+1,+1]\} \equiv \left\{\bar{\mathbf{b}}_1, \bar{\mathbf{b}}_2, \bar{\mathbf{b}}_3, \bar{\mathbf{b}}_4\right\} \tag{20}$$

where $\bar{\mathbf{b}}_q$ represents a possible transmitted symbol vector by user k and is a possible mode of the system, denoted m_q. In effect these possibilities can be represented by four mutually exclusive hypotheses, h_1, h_2, h_3, h_4 each pertaining to a mode,

$$h_q : \mathbf{b}_k = \bar{\mathbf{b}}_q; \text{ referred to as system in mode } m[i] = m_q \tag{21}$$

User k being in a specific mode (say m_q) is associated with a specific transmitted symbol set $\bar{\mathbf{b}}_q$, this 'symbol-mode' association is denoted as $\{\bar{\mathbf{b}}_q, m_q\}$ for the transmitted symbol pair $[b_{k,1}, b_{k,2}]$. This allows symbol detection to be carried out by testing the hypothesis of the system for a specific user k. In general the system mode m_q at any given time for user k will take one of $\mathcal{Q}$ possible values, i.e. $q = 1, \ldots, \mathcal{Q}$ where $\mathcal{Q} = S^{N_T}$. In this case, $\mathcal{Q} = 4$.

The mode representation for the K-user MIMO wireless system as a whole is as follows: For each user there are $\mathcal{Q}$ possible modes and the system of K users will have $\mathcal{Q}^K$ modes (hypotheses). That is the number of modes are exponential in the number of users. A particular mode of the K user system at time-instant i will be M_r where,

$$M_r = [m_{j_{1,r}} \ldots m_{j_{K,r}}], \quad r = 1 \ldots \mathcal{Q}^K \tag{22}$$

r indexes the overall K-user system mode where $j_{k,r}$ is the mode index for user k when system is in mode r. Therefore $j_{k,r} \in \{1, 2, \ldots, \mathcal{Q}\}$ for $k = 1, \ldots, K$. $m_{j_{k,r}}$ will take one of $\mathcal{Q}$ possible values, $\{m_1, m_2, \ldots, m_{\mathcal{Q}}\}$ (as in (21)). Observing the system equations, (18),(19), it is clear that state evolution (18) does not depend on any unknown symbols and hence is the same for all system modes. The measurement (19) differs for each system mode and hence the r-th hypothesis is represented as,

$$\begin{aligned}\mathcal{H}_r : \quad & \mathbf{z}[i] = \mathbf{L}\mathbf{A}\bar{\mathbf{B}}_r\mathbf{c}[i] + \bar{\mathbf{v}}[i], \\ & \text{if system mode at time-instant } i, M[i] = M_r; \text{ (denoted in short as } M_r^i).\end{aligned} \tag{23}$$

which is formed by replacing the transmitted symbol matrix $\mathbf{B}[i]$ of unknown symbols at time-instant i, with the matrix $\bar{\mathbf{B}}_r$ from the 'symbol-mode' association $\{\bar{\mathbf{B}}_r, M_r\}$, where $\mathbf{B}[i] \in \{\ldots \bar{\mathbf{B}}_r \ldots\}$ and $r \in \{1, 2, \ldots, \mathcal{Q}^K\}$. Therefore (18), (23) gives the hybrid system representation pertaining to each possible system mode M_r^i (hypothesis $\mathcal{H}_r$) that a system could be in at a given time.

IV. Multiple Model Estimation for MIMO System

Following the developments in the previous section, the MIMO system is characterized by the state equations, (18), (23), where the combination of continuous states (channel gains) and discrete states (symbols) or modes form a hybrid system.

Consider a MIMO system whose received signal sequence up to time instant i is $\mathbf{Z}^i = [\mathbf{z}[1], \ldots, \mathbf{z}[i]]$. The optimal receiver searches through all possible paths from the time-instant 1 to i given the observations $\mathbf{Z}^i$ of the hybrid system (23). At any time-instant possible combinations of symbols transmitted by K users (or the number of possible modes for the system to be in at that time-instant) is $\mathcal{Q}^K$ (22). Considering a time interval of length i the number of possible sequences (or mode sequences in the hybrid system framework) will be $\mathcal{Q}^{Ki}$ and a specific mode sequence through time-instant i is denoted as $\mathcal{M}_s^i$ for $s \in \{1, \ldots, \mathcal{Q}^{Ki}\}$, and

$$\mathcal{M}_s^i = [M_{j_{1,s}} \ldots M_{j_{i,s}}] \quad s = 1, \ldots, \mathcal{Q}^{Ki} \tag{24}$$

Thus the number of mode sequences to be evaluated grows exponentially in number of users K and sequence length i, making the optimal approach impractical. The s-th sequence of modes through time i is written as, $\mathcal{M}_s^i = [\mathcal{M}_{s'}^{i-1}, M[i]]$ where mode sequence through time-instant $i-1$, denoted as $\mathcal{M}_{s'}^{i-1}$, is its parent sequence and $M[i] = M_r$ its last element.

In the hybrid system (23), if the transmitted symbol sequence is given, the continuous valued state (channel gains) is estimated for the sequence length i, using Bayes' Law and Chapman-Kolmogorov equation,

$$p[\mathbf{c}[i]|\mathcal{M}_s^i, \mathbf{Z}^i] = \frac{p(\mathbf{z}[i]|\mathbf{c}[i], \mathcal{M}_s^i)}{p(\mathbf{z}[i]|\mathcal{M}_s^i, \mathbf{Z}^{i-1})} p(\mathbf{c}[i]|\mathcal{M}_s^i, \mathbf{Z}^{i-1}) \tag{25}$$

$$\text{where,} \quad p(\mathbf{c}[i] \mid \mathcal{M}_s^i, \mathbf{Z}^{i-1}) = \int p(\mathbf{c}[i] \mid \mathbf{c}[i-1], \mathcal{M}_s^i) p(\mathbf{c}[i-1] \mid \mathcal{M}_s^i, \mathbf{Z}^{i-1}) d\mathbf{c}[i-1] \tag{26}$$

and $p(\mathbf{c}[i] \mid \mathbf{c}[i-1], \mathcal{M}_s^i) = p(\mathbf{c}[i] \mid \mathbf{c}[i-1])$ since channel variation is independent of transmitted symbols (18). Therefore, $p(\mathbf{c}[i] \mid \mathbf{c}[i-1]) \sim \mathcal{N}(\mathbf{c}[t]; \mathbf{F}\mathbf{c}[i-1], \sigma_w^2 \mathbf{I}_{KN_TN_R})$ and $p(\mathbf{z}[i]|\mathbf{c}[i], \mathcal{M}_s^i) \sim \mathcal{N}(\mathbf{z}[t]; \mathbf{L}\mathbf{A}\bar{\mathbf{B}}_r\mathbf{c}[i]\ ,\ \rho_v^2 \mathbf{I}_{KLN_TN_R} + \mathbf{L}\mathbf{A}\mathrm{var}(\mathbf{c}[i])\mathbf{A}^\top\mathbf{L}^\top)$ is the mode sequence likelihood given by the Kalman filter innovations likelihood [41]. Since $\mathcal{M}_s^{i-1}$ is the parent sequence of $\mathcal{M}_s^i$, $p(\mathbf{c}[i-1] \mid \mathcal{M}_s^i, \mathbf{Z}^{i-1}) = p(\mathbf{c}[i-1] \mid \mathcal{M}_s^{i-1}, \mathbf{Z}^{i-1})$. For the hybrid system (18),(23), in the event of given transmitted symbols, the above two recursions imply that the channel gain can be estimated by a Kalman filter. When the symbol sequence is unknown, a bank of Kalman filters each matched to all possible symbol sequences result from the above consideration. Demodulation of the transmitted sequence requires the determination of the most likely symbol sequence amongst all possible sequences. The decoded symbol sequence $\hat{\mathcal{B}} \in \{\bar{\mathcal{B}}_s, s \in \{1, 2, \ldots, \mathcal{Q}^{Ki}\}\}$ and Q^{Ki} is the number of system modes (hypotheses). The question then is to determine the probabilities of each mode sequence $\mathcal{M}_s^i$. The maximum a priori (MAP) estimate of the symbol sequence $\hat{\mathcal{B}} = [\hat{\mathbf{B}}[1], \ldots, \hat{\mathbf{B}}[i]]$ is given by,

$$\begin{aligned} \hat{\mathcal{B}} &= \bar{\mathcal{B}}_{s^{MAP}} \\ s^{MAP} &= \arg\max_s P[\,\mathcal{M}_s^i|\mathbf{Z}^i\,] \end{aligned} \tag{27}$$

Therefore the 'symbol-mode' pair association $\{\bar{\mathcal{B}}_{s^{MAP}}, \mathcal{M}^i_{s^{MAP}}\}$ gives the MAP estimate of the symbol matrix sequence $\hat{\mathcal{B}}$. The term $P(\,\mathcal{M}_s^i|\mathbf{Z}^i\,)$ is the conditional probability of the sth mode sequence. The probability of the mode sequence is obtained by Bayes' rule [20],

$$P(\,\mathcal{M}_s^i|\mathbf{Z}^i\,) = \frac{1}{a} p(\mathbf{z}[i]|\mathcal{M}_s^i, \mathbf{Z}^{i-1}) P(\mathcal{M}_s^i|\mathcal{M}_{s'}^{i-1}) P(\,\mathcal{M}_{s'}^{i-1}|\mathbf{Z}^{i-1}\,) \tag{28}$$

where $P(\,\mathcal{M}_{s'}^{i-1}|\mathbf{Z}^{i-1}\,)$ is the conditional probability of the s'th mode sequence for a sequence of length $i-1$, a is the normalization constant. Note that the first probability term on the right side is the marginalized density denominator in (25), the innovation likelihood.

Then, in view of the Markov property (from Assumptions 2.1, 2.2),

$$P(\mathcal{M}_s^i \mid \mathcal{M}_{s'}^{i-1}) = P(M[i] = M_r \mid M[i-1] = M_{r'}) = P(M_r^i|M_{r'}^{i-1}) = \pi_{rr'} \tag{29}$$

where $M_{r'}^{i-1}$ is the last mode in the parent sequence s' through time-instant $i-1$ and $\pi_{r,r'}$ is the mode transition probability. The mode indices $j_{i,s}, j_{i-1,s'}$ have taken the values r, r' respectively and $r, r' \in \{1, \ldots, \mathcal{Q}^K\}$. The transition probability matrix for the system mode is determined from the matrix $[\Pi_0]_{\mathcal{Q}\times\mathcal{Q}}$ and number of users K, which gives $[\Pi_0 \otimes \ldots \otimes \Pi_0]_{\mathcal{Q}^K\times\mathcal{Q}^K}$ where $\pi_{rr'} = [\cdot]_{rr'}$ is a matrix element. The mode transition probability leads to a reduced form of (28),

$$P(\,\mathcal{M}_s^i|\mathbf{Z}^i\,) = \frac{1}{a} p(\mathbf{z}[i]|\mathcal{M}_s^i, \mathbf{Z}^{i-1}) P(M_r^i|M_{r'}^{i-1}) P(\,\mathcal{M}_{s'}^{i-1}|\mathbf{Z}^{i-1}\,) \tag{30}$$

The distribution of the state $\mathbf{c}[i]$ is a weighted mixture of the model-conditioned densities with conditional model probabilities given in (30) as weights,

$$p(\mathbf{c}[i]|\mathbf{Z}^i) = \sum_{s=1}^{\mathcal{Q}^{Ki}} p(\mathbf{c}[i]|\mathcal{M}_s^i, \mathbf{Z}^i) P(\,\mathcal{M}_s^i|\mathbf{Z}^i\,) = \sum_{s=1}^{\mathcal{Q}^{Ki}} \mathcal{N}(\mathbf{c}[t];\, \hat{\mathbf{c}}_s[i|i], \mathbf{\Sigma}_s[i|i]) P(\,\mathcal{M}_s^i|\mathbf{Z}^i\,) \tag{31}$$

The means $\hat{\mathbf{c}}_s[i|i]$, variances $\mathbf{\Sigma}_s[i|i]$ of the state and innovation likelihood can be calculated recursively by a bank of $\mathcal{Q}^{Ki}$ Kalman filters. (25),(27),(30),(31) essentially show a hierarchical structure of state estimation at the inner loop and mode estimation at the outer loop of the computation at time-instant i. The probability of a model representing a particular mode history can be calculated recursively from (30) with respect to a mutually exclusive and exhaustive set of $\mathcal{Q}^{Ki}$ models.

Given that the transmitted symbol sequence defines the mode sequence which in turn influences the channel gain estimates, it is clear that optimal estimation algorithms will incur exponentially increasing computational complexity due to the exponentially increasing number of possible symbol sequences. Thus the number of models, (and thus the filters used) grows exponentially with the number of users K and sequence length i making the optimal solution impractical.

A popular sub-optimal approach is the IMM algorithm [42], [20], which reduces the mode complexity to $\mathcal{Q}^K$ filters by suitable merging of previous estimates and provides a joint estimate of the channel gain and symbol detection. This again is computationally expensive when the number of users and transmit antennae increase in a MIMO system; therefore we develop a successive cancellation based approximation leading to interacting multiple model based successive interference cancellation (IMM-SIC) detector.

V. IMM-SIC for MIMO System

A. IMM Approximation

Implementation of the IMM introduces a hypotheses reduction due to interaction between the state estimates for all the models at the beginning of each filter cycle at time-instant i [42]. Each filter at the start of cycle i is initialized using a weighted mix of model conditioned state estimates from previous filter cycle $i-1$ - mixed initial conditioning. Therefore the number of filters remains the same for every filter cycle. This yields $\mathcal{Q}^K$ filters running in parallel and each filter is matched to a mode.

Approximation 5.1: The mode conditioned prior distribution of the state is approximated such that the past through time instant i is summarized by a mode conditioned estimate and covariance. The weights (mixing probabilities) are prior (predicted) mode probabilities given all the posterior mode probabilities $\{M_{r'}^{i-1}\}_{r'=1}^{\mathcal{Q}^K}$ from the previous cycle. The mode-conditioned posterior probability density function (PDF),

$$p(\mathbf{c}[i]|M_r^i, \mathbf{Z}^i) = \frac{p(\mathbf{z}[i]|M_r^i, \mathbf{c}[i])}{p(\mathbf{z}[i]|M_r^i, \mathbf{Z}^{i-1})} p(\mathbf{c}[i]|M_r^i, \mathbf{Z}^{i-1}) \tag{32}$$

is essentially the same as (25) but conditioned with modes at time-instant i, rather than the sequence of modes $\mathcal{M}_s^i$. The prior is,

$$p(\mathbf{c}[i]|M_r^i, \mathbf{Z}^{i-1}) = \sum_{r'=1}^{\mathcal{Q}^K} p(\mathbf{c}[i]|M_r^i, M_{r'}^{i-1}, \mathbf{Z}^{i-1}) P(M_{r'}^{i-1}|M_r^i, \mathbf{Z}^{i-1}) \tag{33}$$

$$\approx \sum_{r'=1}^{\mathcal{Q}^K} p(\mathbf{c}[i]|M_r^i, M_{r'}^{i-1}, \hat{\mathbf{c}}_{r'}[i-1|i-1], \Sigma_{r'}[i-1|i-1]) \mu_{r|r'}[i] \tag{34}$$

Similar to (26) the prior distribution $p(\mathbf{c}[i]|M_r^i, M_{r'}^{i-1}, \mathbf{Z}^{i-1})$ (33) will follow from the Chapman-Kolmogorov equation. The approximation is reflected in the second line of (34) where in the first term on the R.H.S. the filters' means and covariances are used as sufficient statistics of all past observations. The prior mode probabilities are $\mu_{r|r'}[i] = P(M_{r'}^{i-1}|M_r^i, \mathbf{Z}^{i-1})$. Therefore instead of considering a mode sequence, we just consider only the mode and associated state estimates at the previous time-instant. The expression (34) is of the same form as (31) in the sense that it is still a mixture normal distribution but the mean and covariance of each normal distribution are computed differently and at each time-instant using prior estimates from the previous time-instant. The decoded symbol matrix $\hat{\mathbf{B}} \in \{\bar{\mathbf{B}}_r, r \in \{1, 2, \ldots, \mathcal{Q}^K\}\}$ and $\mathcal{Q}^K$ is the number of system modes (hypotheses). The MAP estimate of symbol matrix $\hat{\mathbf{B}}[i] = \text{diag}(\hat{\mathbf{B}}_1[i], \ldots, \hat{\mathbf{B}}_K[i])$ is given by

$$\begin{aligned} \hat{\mathbf{B}}[i] &= \bar{\mathbf{B}}_{r^{MAP}} \\ r^{MAP} &= \arg\max_r P[\, M_r^i|\mathbf{Z}^i\,] \end{aligned} \tag{35}$$

Therefore the 'symbol-mode' pair association $\{\bar{\mathbf{B}}_{r^{MAP}}, M_{r^{MAP}}^i\}$ gives the MAP estimate of the transmitted symbol matrix $\hat{\mathbf{B}}[i]$. The term $\mu_r[i] = P(\, M_r^i|\mathbf{Z}^i\,)$ is the conditional posterior probability of the rth mode which is obtained by [20],

$$P(\, M_r^i|\mathbf{Z}^i\,) = \frac{1}{a} p(\mathbf{z}[i]|M_r^i, \mathbf{Z}^{i-1}) \sum_{r'=1}^{\mathcal{Q}^K} P(M_r^i|M_{r'}^{i-1}) P(\, M_{r'}^{i-1}|\mathbf{Z}^{i-1}\,) \tag{36}$$

where $\mu_{r'}[i-1] = P(\, M_{r'}^{i-1}|\mathbf{Z}^{i-1}\,)$ is the conditional posterior probability of the r'th mode at $i-1$, a is the normalization constant. Note that the first probability term on the right side is the marginalized density denominator in (32), the innovation likelihood. At time i, predictive state estimates are computed under each possible current mode conditioned on all previous $\mathcal{Q}^K$ modes (34). The posterior channel gains are then calculated (32) by $\mathcal{Q}^K$ set of filters, each filter conditioned on each possible current mode. The aggregated channel estimate at a given time is calculated from a mixture Gaussian of $\mathcal{Q}^K$ terms,

$$p(\mathbf{c}[i]|\mathbf{Z}^i) \approx \sum_{r=1}^{\mathcal{Q}^K} p(\mathbf{c}[i]|M_r^i, \mathbf{Z}^i) P(M_r^i|\mathbf{Z}^i) \tag{37}$$

The state estimates adapt with time and $M_{r^{MAP}}^i$ represents the mode (associated with the transmitted symbol set of K users) the system is in at current time and the IMM cycle is repeated recursively. The complexity of the IMM based online joint symbol detector and channel estimator for a K user MIMO system presented above is still exponential in number of users. Further approximation based on successive cancellation within the probabilistic framework is however possible.

B. SIC Approximation

Consider the signal model (17) developed in Section II. For the kth user,

$$\mathbf{z}_k[i] \;=\; \sum_{k'=1}^{k} \mathbf{L}_{k,k'} \mathbf{A}_{k'} \mathbf{B}_{k'}[i] \mathbf{c}_{k'}[i] + \bar{\mathbf{v}}_k[i]$$

Since $\mathbf{L}$ in (17) is a lower triangular matrix, the matched noise-whitened output for user k is a combination of multi-user interference (MUI) term of all users up to $k-1$ and the users signal

$$\mathbf{z}_k[i] = \underbrace{\sum_{k'=1}^{k-1} \mathbf{L}_{k,k'} \mathbf{A}_{k'} \mathbf{B}_{k'}[i] \mathbf{c}_{k'}[i]}_{MUI} + \mathbf{L}_{k,k} \mathbf{A}_k \mathbf{B}_k[i] \mathbf{c}_k[i] + \bar{\mathbf{v}}_k[i] \tag{38}$$

This can be expressed in a matrix form,

$$\mathbf{z}_k[i] = \mathbf{L}_{k,1:k-1} \mathbf{A}_{1:k-1} \mathbf{B}_{1:k-1}[i] \mathbf{c}_{1:k-1}[i] + \mathbf{L}_{k,k} \mathbf{A}_k \mathbf{B}_k[i] \mathbf{c}_k[i] + \bar{\mathbf{v}}_k[i] \tag{39}$$

where $\mathbf{L}_{k,1:k-1} = [\mathbf{L}_{k,1}, \ldots, \mathbf{L}_{k,k-1}], \mathbf{B}_{1:k-1}[i] = \text{diag}(\mathbf{B}_1[i], \ldots, \mathbf{B}_{k-1}[i]), \mathbf{A}_{1:k-1} = \text{diag}(\mathbf{A}_1, \ldots, \mathbf{A}_{k-1}), \mathbf{c}_{1:k-1}[i] = [\mathbf{c}_1[i]^H, \ldots, \mathbf{c}_{k-1}[i]^H]^H$.

Approximation 5.2 IMM-SIC Algorithm: The IMM approximation (5.1), where the past through time instant i is summarised at a single time instant, considers all users collectively. The basis for the SIC approximation is for the detector to consist of a filter/detector for each user which utilizes information from only the previous $(k-1)$ users and the current user's information. The unknown discrete mode at time-instant i for user k, i.e. $m[i] = m_q$, is denoted in short as $m^i_{q_k}$ where $k = 1, \ldots, K$; each mode is decoded successively $\forall k$. Unlike in the previous section, here a mode $m^i_{q_k}$ represents the discrete state of each user and not the discrete state of the overall (K) users' system M^i_r, at a time-instant i. A mode sequence $\{m^i_{q_1}, \ldots, m^i_{q_k}\}$ is denoted as $m^i_{q_{1:k}}$ where the indices $q_k \in \{1, \ldots, \mathcal{Q}\}$. The posterior conditional density for current user k is given from Bayes Law,

$$p(\mathbf{c}_k[i] | m^i_{q_k}, \mathbf{Z}^i_{1:k}) = \frac{p(\mathbf{z}_k[i] | \mathbf{c}_k[i], m^i_{q_k}, \mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i])}{p(\mathbf{z}_k[i] | m^i_{q_k}, \mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i])} p(\mathbf{c}_k[i] | m^i_{q_k}, \mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i]) \tag{40}$$

which is essentially the same as (32) but for user k only conditioned on kth user's modes at time-instant i. Note that the distributions on the R.H.S. all conditioned on $\mathbf{Z}^{i-1}_{1:k} = [\mathbf{z}_{1:k}[1], \ldots, \mathbf{z}_{1:k}[i-1]]$ and $\mathbf{z}_{1:k-1}[i] = [\mathbf{z}_1[i], \ldots, \mathbf{z}_{k-1}[i]]$. The prior term in (40) is given as,

$$p(\mathbf{c}_k[i] | m^i_{q_k}, \mathbf{Z}^{i-1}_{1:k}) = \sum_{q'_k=1}^{\mathcal{Q}} p(\mathbf{c}_k[i] | m^i_{q_k}, m^{i-1}_{q'_k}, \mathbf{Z}^{i-1}_{1:k}) P(m^{i-1}_{q'_k} | m^i_{q_k}, \mathbf{Z}^{i-1}_{1:k}) \tag{41}$$

$$\approx \sum_{q'_k=1}^{\mathcal{Q}} p(\mathbf{c}_k[i] | m^i_{q_k}, m^{i-1}_{q'_k}, \hat{\mathbf{c}}_{q',k}[i-1|i-1], \hat{\boldsymbol{\Sigma}}_{q',k}[i-1|i-1]) \mu_{q|q'}[i] \tag{42}$$

Similar to (34) the term $p(\mathbf{c}_k[i] | m^i_{q_k}, m^{i-1}_{q'_k}, \mathbf{Z}^{i-1}_{1:k})$ (41) will follow from Chapman-Kolmogorov equation and the past observations are approximated by the second order statistics of each filter of user k from the previous time-instant (42). Having dealt with the prior in (40) consider the likelihood

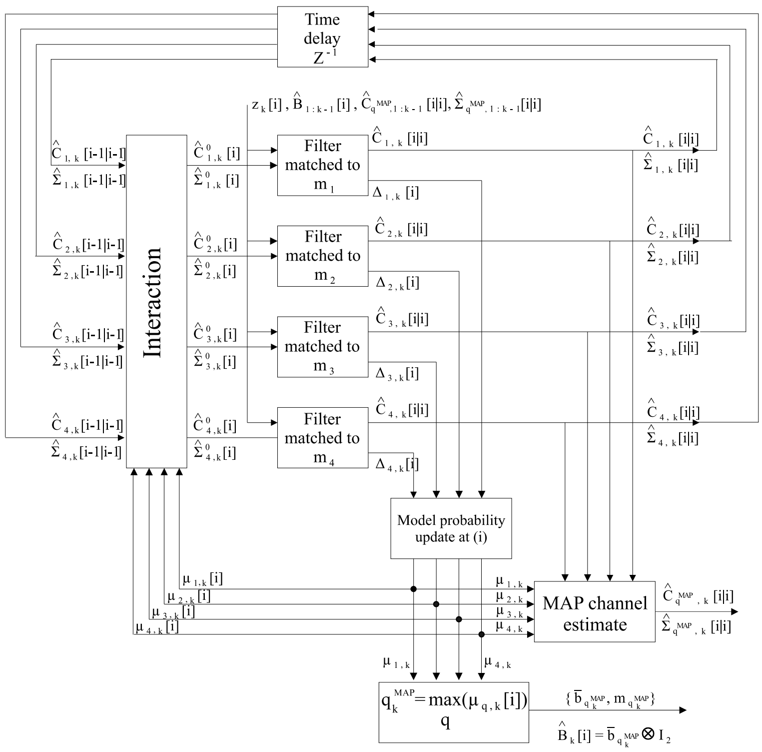

Fig. 2. IMM-SIC cycle for user k symbol detection and channel estimation.

term on R.H.S. which is expressed as,

$$p(\mathbf{z}_k[i]|\mathbf{c}_k[i], m^i_{q_k}, \mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i]) = \int \sum p(\mathbf{z}_k[i]|\mathbf{c}_{1:k}[i], m^i_{q_{1:k}}, \mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i]) \\ p(\mathbf{c}_{1:k-1}[i]|m^i_{q_{1:k-1}}, \mathbf{Z}^i_{1:k-1}) P(m^i_{q_{1:k-1}}|\mathbf{Z}^i_{1:k-1}) d\mathbf{c}_{1:k-1}[i] \qquad (43)$$

The discrete summation is over all possible $\mathcal{Q}^{k-1}$ mode sequences and the integration marginalizes over the previous $k-1$ users channel estimators. The first step in simplifying the likelihood is that the joint distribution of mode probabilities are approximately decomposed as,

$$P(m^i_{q_{1:k-1}}|\mathbf{Z}^i_{1:k-1}) = P(m^i_{q_{1:k-2}}|\mathbf{Z}^i_{1:k-1})P(m^i_{q_{k-1}}|\mathbf{Z}^i_{1:k-1}) \approx P(m^i_{q_{1:k-2}}|\mathbf{Z}^i_{1:k-2})P(m^i_{q_{k-1}}|\mathbf{Z}^i_{1:k-1}) \quad (44)$$

Each user k considers only the information of its own and $(k-1)$ previous users up to current time for symbol detection which is reflected in the approximation in the first term above. Applying the same successively will lead to

$$P(m^i_{q_{1:k-1}}|\mathbf{Z}^i_{1:k-1}) \approx \prod_{\kappa=1}^{k-1} P(m^i_{q_\kappa}|\mathbf{Z}^i_{1:\kappa}) \qquad (45)$$

The 'symbol-mode' association is $\{\bar{\mathbf{b}}_q, m_q\}$ (refer to (20)). The decoded symbol matrix of user κ is $\hat{\mathbf{B}}_\kappa \in \{\bar{\mathbf{b}}_q \otimes \mathbf{I}_{N_R},\ q \in \{1, 2, \ldots, \mathcal{Q}\}\}$ and $\mathcal{Q}$ is the number of system modes (hypotheses). The

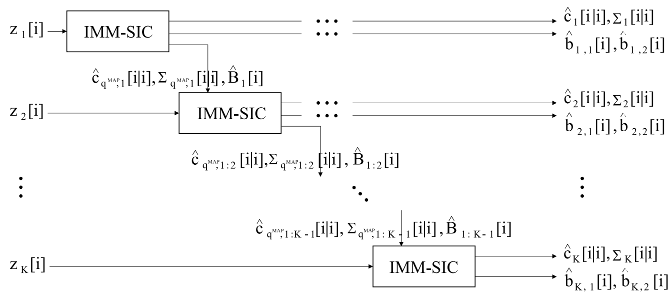

Fig. 3. Diagrammatic representation of the Multiuser IMM-SIC Detector. Each block is an IMM-SIC filter.

MAP estimate of user symbols of κth user at time-instant i will be,

$$\begin{aligned} \hat{\mathbf{B}}_\kappa[i] &= \bar{\mathbf{b}}_{q_\kappa^{MAP}} \otimes \mathbf{I}_{N_R} \\ \text{where} \quad q_\kappa^{MAP} &= \arg\max_q P(m[i] = m_q | \mathbf{Z}_{1:\kappa}^i) \end{aligned} \tag{46}$$

Having approximated the probability densities such that detection for user k utilizes information from only the first k users, we still have a mixture density with $\mathcal{Q}^k$ components. In order to reduce this complexity, the previous users' mode probabilities, $P(m_{q_\kappa}^i|\mathbf{Z}_{1:\kappa}^i)$ for $\kappa = 1, \dots, k-1$, are approximated by the MAP estimate $m_{q_\kappa^{MAP}}^i$ (46) of each user. Therefore for the k-th user's estimation cycle,

$$P(m_{q_\kappa}^i|\mathbf{Z}_{1:\kappa}^i) \approx \begin{cases} 1 & \text{if} \;\; m_{q_\kappa}^i = m_{q_\kappa^{MAP}} \\ 0 & \text{if} \;\; m_{q_\kappa}^i \neq m_{q_\kappa^{MAP}} \end{cases} \qquad \text{for } q = 1, \dots, \mathcal{Q} \text{ and } \kappa = 1, \dots, k-1 \tag{47}$$

Substituting (47) in (43) leads to,

$$\begin{aligned} p(\mathbf{z}_k[i]|\mathbf{c}_k[i], m_{q_k}^i, \mathbf{Z}_{1:k}^{i-1}, \mathbf{z}_{1:k-1}[i]) = \int p(\mathbf{z}_k[i]|\mathbf{c}_{1:k}[i], m_{q_k}^i, m_{q_{1:k-1}^{MAP}}^i, \mathbf{Z}_{1:k}^{i-1}, \mathbf{z}_{1:k-1}[i]) \\ p(\mathbf{c}_{1:k-1}[i]|m_{q_{1:k-1}^{MAP}}^i, \mathbf{Z}_{1:k-1}^i) d\mathbf{c}_{1:k-1}[i] \end{aligned} \tag{48}$$

The integration is now over a single sequence composed of the MAP modes of $(k-1)$ users and associated channel gains. In general consider the joint MAP distribution of a set of (k) users, $p(\mathbf{c}_{1:k}[i]|m_{q_{1:k}}^i, \mathbf{Z}_{1:k}^i)$, a similar approximation to (45) can be made.

$$\begin{aligned} p(\mathbf{c}_{1:k}[i]|m_{q_{1:k}^{MAP}}^i, \mathbf{Z}_{1:k}^i) &= p(\mathbf{c}_{1:k-1}[i]|\mathbf{c}_k[i], m_{q_{1:k}^{MAP}}^i, \mathbf{Z}_{1:k}^i) p(\mathbf{c}_k[i]|m_{q_{1:k}^{MAP}}^i, \mathbf{Z}_{1:k}^i) \\ &\approx p(\mathbf{c}_{1:k-1}[i]|m_{q_{1:k-1}^{MAP}}^i, \mathbf{Z}_{1:k-1}^i) p(\mathbf{c}_k[i]|m_{q_{1:k}^{MAP}}^i, \mathbf{Z}_{1:k}^i) \\ &\approx \prod_{\kappa=1}^{k} p(\mathbf{c}_\kappa[i]|m_{q_{1:\kappa}^{MAP}}^i, \mathbf{Z}_{1:\kappa}^i) \end{aligned} \tag{49}$$

The individual terms in the product on R.H.S. is still conditioned on all k users' MAP modes.

TABLE II

One Cycle of IMM-SIC estimator and detector for MIMO system at time-instant (i) for MS k

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Consider the following BPSK system (for $q = 1, 2, \ldots, \mathcal{Q}$):

$$\mathbf{c}_{q,k}[i+1] = \mathbf{F}_k \mathbf{c}_{q,k}[i] + \mathbf{w}_k[i]$$

$$\mathbf{z}_k[i] = \underbrace{\mathbf{L}_{k,k}\mathbf{A}_k[\bar{\mathbf{b}}_q \otimes \mathbf{I}_{N_R}]}_{\mathbf{H}_{q,k}[i]} \mathbf{c}_k[i] + \mathbf{V}_k[i] + \bar{\mathbf{v}}_k[i]$$

with $\mathbf{c}_{q,k}[0] \sim \mathcal{N}[\bar{\mathbf{c}}_0, \bar{\mathbf{\Sigma}}_0]$, $\mathbf{F} = \mathrm{diag}(\mathbf{F}_1, \ldots, \mathbf{F}_K)$. Mode $m^i_{q_k}$ represents a possible symbol pair (a hypothesis of the system) transmitted by user k at time-instant i.

1) Interaction/Mixing of the estimates (for $q = 1, 2, \ldots, \mathcal{Q}$):
   mixing probability: $\mu_{q|q'}[i] = \pi_{qq'}\mu_{q',k}[i-1] / \sum_{q'=1}^{\mathcal{Q}} \pi_{qq'}\mu_{q',k}[i-1]$
   mixing estimate: $\hat{\mathbf{c}}^0_{q,k}[i] = \sum_{q'=1}^{\mathcal{Q}} [\hat{\mathbf{c}}_{q',k}[i-1]]\mu_{q|q'}[i]$
   mixing covariance:
   $$\mathbf{\Sigma}^0_{q,k}[i] = \sum_{q'=1}^{\mathcal{Q}} \Big[\mathbf{\Sigma}_{q',k}[i-1] + \big[\hat{\mathbf{c}}^0_{q,k}[i] - \hat{\mathbf{c}}_{q',k}[i-1]\big]\,\big[\hat{\mathbf{c}}^0_{q,k}[i] - \hat{\mathbf{c}}_{q',k}[i-1]\big]^\top\Big]\ \mu_{q|q'}[i]$$
2) Mode-conditioned filtering (for $q = 1, \ldots\ldots \mathcal{Q}$):
   predicted state from $i-1$ to i: $\hat{\mathbf{c}}_{q,k}[i|i-1] = \mathbf{F}_k \hat{\mathbf{c}}^0_{q,k}[i]$
   predicted covariance: $\mathbf{\Sigma}_{q,k}[i|i-1] = \mathbf{H}_{q,k}\mathbf{\Sigma}^0_{q,k}[i]\mathbf{H}^\top_{q,k} + \rho^2_w \mathbf{I}_{N_T N_R}$
   measurement residual (55): $\epsilon_{q,k} = \mathbf{z}_k[i] - \mathbf{H}_{q,k}\hat{\mathbf{c}}_{q,k}[i|i-1] - \bar{\mathbf{V}}_k[i]$
   residual covariance (55): $\mathbf{S_{q,k}} = \mathbf{H}_{q,k}\mathbf{\Sigma}_{q,k}[i|i-1]\mathbf{H}^\top_{q,k} + \mathrm{Var}(\mathbf{V}_k[i]) + \rho^2_v \mathbf{I}_{N_T N_R}$
   filter gain: $\mathbf{K}_{q,k} = \mathbf{\Sigma}_{q,k}[i|i-1]\mathbf{H}^\top_{q,k}\mathbf{S_{q,k}}^{-1}$
   updated state : $\hat{\mathbf{c}}_{q,k}[i|i] = \hat{\mathbf{c}}_{q,k}[i|i-1] + \mathbf{K}_{q,k}\epsilon_{q,k}$
   updated covariance: $\mathbf{\Sigma}_{q,k}[i|i] = \mathbf{\Sigma}_{q,k}[i|i-1] - \mathbf{K}_{q,k}\mathbf{S_{q,k}}\mathbf{K}^\top_{q,k}$
   likelihood function: $\Delta_{q,k}[i] = \frac{1}{\sqrt{|(2\pi)\mathbf{S_{q,k}}|}}\exp[-\frac{1}{2}\epsilon^\top_{q,k}\mathbf{S_{q,k}}^{-1}\epsilon_{q,k}]$
3) Mode probability updates and bit detection:
   mode probability: $\mu_{q,k}[i] = \frac{1}{c}\Delta_{q,k}[i] \sum_{q'=1}^{\mathcal{Q}} \pi_{qq'}\mu_{q',k}[i-1]$
   decision rule: $q^{MAP}_k = \max_q \mu_{q,k}[i]$ and $\hat{\mathbf{B}}_k[i] = \bar{\mathbf{b}}_{q^{MAP}_k} \otimes \mathbf{I}_{N_R}$; $[\hat{\beta}_{k,0}[i], \ldots, \hat{\beta}_{k,N_s}[i]] = \bar{\mathbf{b}}_{q^{MAP}_k}$
4) combination of estimates:
   aggregated estimate : $\hat{\mathbf{c}}_k[i] = \sum_{q=1}^{\mathcal{Q}} [\hat{\mathbf{c}}_{q,k}[i|i]]\mu_{q,k}[i]$
   aggregated covariance: $\mathbf{\Sigma}_k[i] = \sum_{q=1}^{\mathcal{Q}} [\mathbf{\Sigma}_{q,k}[i|i] + [\hat{\mathbf{c}}_k[i] - \hat{\mathbf{c}}_{q,k}[i|i]][\hat{\mathbf{c}}_k[i] - \hat{\mathbf{c}}_{q,k}[i|i]]^\top]\mu_{q,k}[i]$

This can be simplified further. Consider an individual user κ's state distribution conditioned on its own MAP mode,

$$p(\mathbf{c}_\kappa[i]|m^i_{q^{MAP}_\kappa}, \mathbf{Z}^i_{1:\kappa}) = \sum p(\mathbf{c}_\kappa[i]|m^i_{q^{MAP}_\kappa}, m^i_{q_{1:\kappa-1}}, \mathbf{Z}^i_{1:\kappa}) P(m^i_{q_{1:\kappa-1}}|\mathbf{Z}^i_{1:\kappa-1}) \quad (50)$$

The summation is over all possible mode sequences $\mathcal{Q}^{\kappa-1}$. From (45) and substituting for the MAP estimates of previous users (from (47)) approximates the above to,

$$p(\mathbf{c}_\kappa[i]|m^i_{q^{MAP}_\kappa}, \mathbf{Z}^i_{1:\kappa}) \approx p(\mathbf{c}_\kappa[i]|m^i_{q^{MAP}_\kappa}, m^i_{q^{MAP}_{1:\kappa-1}}, \mathbf{Z}^i_{1:\kappa}) = p(\mathbf{c}_\kappa[i]|m^i_{q^{MAP}_{1:\kappa}}, \mathbf{Z}^i_{1:\kappa}) \quad (51)$$

Noting the approximate equivalence above, (49) can be written as,

$$p(\mathbf{c}_{1:k}[i]|m^i_{q^{MAP}_{1:k}}, \mathbf{Z}^i_{1:k}) \approx \prod_{\kappa=1}^{k} p(\mathbf{c}_\kappa[i]|m^i_{q^{MAP}_\kappa}, \mathbf{Z}^i_{1:\kappa}) \quad (52)$$

essentially a product of the individual users' MAP mode conditioned state distribution. This completes the approximate decomposition. Substituting the above approximation in (48) gives,

$$p(\mathbf{z}_k[i]|\mathbf{c}_k[i], m^i_{q_k}, \mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i]) \approx \int p(\mathbf{z}_k[i]|\mathbf{c}_{1:k}[i], m^i_{q_k}, m^i_{q^{MAP}_{1:k-1}}, \mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i]) \prod_{\kappa=1}^{k-1} p(\mathbf{c}_\kappa[i]|m^i_{q^{MAP}_\kappa}, \mathbf{Z}^i_{1:\kappa}) d\mathbf{c}_\kappa[i] \quad (53)$$

The term $p(\mathbf{z}_k[i]|\mathbf{c}_{1:k}[i], m^i_{q_{1:k}}, \mathbf{Z}^i_{1:k-1})$ is obtained from (39). With the prior assumed Gaussian and with the approximations made above, the posterior distribution of the channel for user k, $p(\mathbf{c}_k[i]|m^i_{q^{MAP}_k}, \mathbf{Z}^i_{1:k})$ is a Gaussian distribution and is expressed as,

$$p(\mathbf{c}_k[i]|m^i_{q^{MAP}_k}, \mathbf{Z}^i_{1:k}) \sim \mathcal{N}(\mathbf{c}_k[t]; \hat{\mathbf{c}}_{q^{MAP},k}[i|i], \mathbf{\Sigma}_{q^{MAP},k}[i|i], m_{q^{MAP}_k}) \quad (54)$$

Note that the mean $\hat{\mathbf{c}}_{q^{MAP},k}[i|i]$ and variance $\mathbf{\Sigma}_{q^{MAP},k}[i|i]$ will be given by the MAP mode filter for user κ. The approximations reflect the fact that channel estimation for user k use only observations up to k, even if we have observations up to user K, and that the estimated symbols are treated as the true transmitted symbols. This is essentially a decision feedback process that approximates the MUI and noise as a single Gaussian distribution. From being a mixture distribution substituting (54) in (53) and performing the marginalization leads to the distribution on the L.H.S. of (53) being approximated as a single Gaussian with,

$$\begin{aligned} p(\mathbf{z}_k[i]|\mathbf{c}_k[i], m^i_{q_k}, \mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i]) \quad \sim \quad & \mathcal{N}(\mathbf{z}_k[t]; \mathbf{H}_{q,k}[i]\mathbf{c}_k[i] + \bar{\mathbf{V}}_k[i] \ , \ \mathbf{H}_{q,k}[i]\Sigma_k[i]\mathbf{H}_{q,k}[i]^\top \\ & + \mathrm{Var}\{\mathbf{V}_k[i]|\mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i]\} + \rho_v^2 \mathbf{I}_{N_T N_R}) \end{aligned} \quad (55)$$

where,

$$\begin{aligned} \mathbf{H}_{q,k}[i] &= \mathbf{L}_{k,k}\mathbf{A}_k[\bar{\mathbf{b}}_q \otimes \mathbf{I}_{N_R}]; \text{ and } \bar{\mathbf{V}}_k[i] = \mathbf{H}_{k,1:k-1}[i]\hat{\mathbf{c}}_{q^{MAP},1:k-1}[i|i] \\ \mathrm{Var}\{\mathbf{V}_k[i]|\mathbf{Z}^{i-1}_{1:k}, \mathbf{z}_{1:k-1}[i]\} &= \mathbf{H}_{k,1:k-1}[i]\Sigma_{q^{MAP},1:k-1}[i|i]\mathbf{H}_{k,1:k-1}[i]^\top \\ \mathbf{H}_{k,1:k-1} &= \mathbf{L}_{k,1:k-1}\mathbf{A}_{1:k-1}\hat{\mathbf{B}}_{1:k-1}[i] \\ \hat{\mathbf{c}}_{q^{MAP},1:k-1}[i|i] &= [\hat{\mathbf{c}}_{q^{MAP},1}[i|i]^H, \ldots, \hat{\mathbf{c}}_{q^{MAP},k-1}[i|i]^H]^H \\ \Sigma_{q^{MAP},1:k-1}[i|i] &= \mathrm{diag}(\Sigma_{q^{MAP},1}[i|i], \ldots, \Sigma_{q^{MAP},k-1}[i|i]) \end{aligned} \quad (56)$$

Equivalently the qth hypothesis at k-th recursion of the IMM-SIC filter becomes,

$$\mathbf{z}_k[i] = \mathbf{L}_{k,k}\mathbf{A}_k[\bar{\mathbf{b}}_q \otimes \mathbf{I}_{N_R}]\mathbf{c}_k[i] + \mathbf{V}_k[i] + \bar{\mathbf{v}}_k[i] \tag{57}$$

where $\mathbf{V}_k[i]$ is approximated as a Gaussian distributed random variable, $\mathbf{V}_k[i] \sim \mathcal{N}(\mathbf{V}_k[i]; \bar{\mathbf{V}}_k[i]$, $\text{Var}(\mathbf{V}_k[i]))$, which gives the MUI term. The IMM-SIC estimation cycle is repeated recursively for each user. Therefore for user k there will be $\mathcal{Q}$ parallel filters each matched to a mode $m^i_{q_k}$ (see Fig. 2) and the weights associated with each mode are the posterior mode probabilities $P(m^i_{q_k}|\mathbf{Z}^i_{1:k})$ (46). See Table II for algorithmic details. Fig. 3 gives a diagrammatic representation of the IMM-SIC decoder for a K-user MIMO system. The IMM-SIC detector does not rely on the ordering of users based on the received powers. While it is applicable to a MIMO system, it can be extended to a ST coded system as well.

C. IMM-SIC for MIMO STC System

Considering an Alamouti ST coded system, at time $2i$ the received signal (13) will be,

$$r[2i] = \sum_{k=1}^{K} \mathbf{A}_k \begin{bmatrix} \mathbf{S}^0_{k,1} & \mathbf{S}^1_{k,1} \end{bmatrix} \begin{bmatrix} b_{k,1}[2i]\mathbf{I}_{N_R} & 0 \\ 0 & b_{k,2}[2i]\mathbf{I}_{N_R} \end{bmatrix} \begin{bmatrix} \mathbf{c}^1_{k,1}[2i] \\ \mathbf{c}^2_{k,1}[2i] \end{bmatrix} + \mathbf{v}[2i] \tag{58}$$

for a 2×2 MIMO system with $L = 1$ and ST coding (1) where each transmission stream n_s has an independent spreading sequence. The received signal $\mathbf{r}[2i]$ is matched filtered (16) and noise whitened (17) to yield $\mathbf{z}[2i]$ with transition matrix Π_0 used to compute mixing probabilities. Similarly at time $2i + 1$,

$$r[2i+1] = \sum_{k=1}^{K} \mathbf{A}_k \begin{bmatrix} \mathbf{S}^1_{k,1} & \mathbf{S}^0_{k,1} \end{bmatrix} \begin{bmatrix} b_{k,1}[2i+1]\mathbf{I}_{N_R} & 0 \\ 0 & b_{k,2}[2i+1]\mathbf{I}_{N_R} \end{bmatrix} \begin{bmatrix} \mathbf{c}^1_{k,1}[2i+1] \\ \mathbf{c}^2_{k,1}[2i+1] \end{bmatrix} + \mathbf{v}[2i+1] \tag{59}$$

the received signal $\mathbf{r}[2i+1]$ is matched filtered and noise whitened to yield $\mathbf{z}[2i+1]$ with transition probability matrix Π_1 (refer to Table I) used to compute mixing probabilities. The IMM-SIC decoder for STC system is given in Table III. By posing the evolution of discrete states of the MIMO ST coded system as a Markov transition we are able to call the IMM-SIC recursively at step $2i$ and $2i + 1$, using transition probability matrices Π_0 and Π_1 (see Fig. 4), latter with probabilities which is a reflection of the symbol level ST coding at the transmitter. The channel estimate and variance is tracked during time-instant $2i$ and $2i+1$ and at each time $\mathbf{V}_{k+1}$ is updated for user $k + 1$. At time $2i + 1$ the original bit sequence $\{\beta_{k,n_s}[i]\}^{N_s}_{n_s=0}$ is decoded.

1) Computational complexity: The number of multiplications required per iteration for each user of the IMM-SIC detector is analysed here. In the following table $n = LN_RN_T$ represents the total number of elements in the state vector and a reciprocal operation is equivalent to three

SIC Steps for each user	Number of multiplications
$\mathbf{H}_{k,1:k-1}[i] = \mathbf{L}_{k,1:k-1}\mathbf{A}_{1:k-1}\hat{\mathbf{B}}_{1:k-1}[i]$	$(k-1)n^2 + (k-1)n \equiv O(n^2)$
SIC Steps for each measurement component/user	
$\bar{\mathbf{h}} = [\mathbf{H}_{k,1:k-1}[i]]_{(n_t-1)LN_R+(n_r-1)L+l,1:(k-1)n}$	0
$[\bar{\mathbf{V}}_k[i]]_l^{n_t,n_r} = \bar{\mathbf{h}}\hat{\mathbf{c}}_{q_{MAP},1:k-1}[i\|i]$	$(k-1)n$
$[Var(\bar{\mathbf{V}}_k[i])]_l^{n_t,n_r} = \bar{\mathbf{h}}\boldsymbol{\Sigma}_{q_{MAP},1:k-1}[i\|i]\bar{\mathbf{h}}^\top$	$(k-1)n + (k-1)n^2$
Total operations count	$2(k-1)n + (k-1)n^2 \equiv O(n^2)$
Kalman filter steps for each mode/meas. component/user	
$\mathbf{h}_{k,l}^{n_t,n_r} = [\mathbf{H}_{q,k}[i]]_{(n_t-1)LN_R+(n_r-1)L+l,1:n}$	$n^2 + n$
$\boldsymbol{\Sigma}_{q,k}[i\|i-1] = \boldsymbol{\Sigma}_{q,k}^0[i] + \rho_w^2$	0
$\mathbf{D}_l^{n_t,n_r} = \boldsymbol{\Sigma}_{q,k}[i\|i-1]\mathbf{h}_{k,l}^{n_t,n_r}$	n^2
$\mathbf{k}_l^{n_t,n_r} = \mathbf{D}_l^{n_t,n_r}/[\mathbf{h}_{k,l}^{n_t,n_r\top}\mathbf{D}_l^{n_t,n_r} + [Var(\bar{\mathbf{V}}_k[i])]_l^{n_t,n_r} + \rho_v^2]$	$n+3$
$\boldsymbol{\Sigma}_{q,k}[i\|i] = \boldsymbol{\Sigma}_{q,k}[i\|i-1] - \mathbf{k}_l^{n_t,n_r}\mathbf{D}_l^{n_t,n_r\top}$	n^2
$\epsilon_l^{n_t,n_r} = z_{k,l}^{(n_t,n_r)} - \mathbf{h}_{k,l}^{n_t,n_r\top}\hat{\mathbf{c}}_{q,k}[i-1\|i-1] - [\bar{\mathbf{V}}_k[i]]_l^{n_t,n_r}$	n
$\hat{\mathbf{c}}_{q,k}[i\|i] = \hat{\mathbf{c}}_{q,k}^0[i] + \mathbf{k}_l^{n_t,n_r}\epsilon_l^{n_t,n_r}$	n
Total operations count	$3n^2 + 4n + 3 \equiv O(n^2)$

multiplications. With $\mathcal{Q}$ modes in total, the operations count per measurement component per user is $O(\mathcal{Q}n^2 + n^2) \equiv O(\mathcal{Q}n^2)$ and with n measurement components for each user, the total operations count per user is $O(\mathcal{Q}n^3 + n^2) \equiv O(\mathcal{Q}n^3)$. Therefore for a K user MIMO system the complexity at each time is $O(K\mathcal{Q}[LN_RN_T]^3)$ linear in number of users K and number of modes $\mathcal{Q} = S^{N_T}$.

The IMM filter based decoder (Approximation 5.1) has $\mathcal{Q}^K$ models (number of filters) at each time instant and the computational complexity will be $O(\mathcal{Q}^K(KLN_RN_T)^3)$, exponential in number of users K. The optimal multiple model decoder will have $\mathcal{Q}^{Ki}$ models (number of filters) at each time instant and the complexity will be $O(\mathcal{Q}^{Ki}(KLN_RN_T)^3)$, exponential in number of users K and time period i.

VI. Simulations

1) IMM-SIC symbol detector for MIMO system: The performance of IMM-SIC is analyzed through computer simulations. We consider a three-user ($K = 3$) flat-fading CDMA channel with processing gain of $N = 31$. Each transmitter is assigned an independent signature waveform. The user-spreading waveform matrix $\mathbf{S}$ is generated randomly from a set of gold sequences. Each user has two transmitters ($N_T = 2$) with each i.i.d. sequence transmitted from each antenna arriving via L paths at each receiver ($N_R = 2$). The i.i.d. sequence is considered a special case of a sequence from a Markov source and all elements of the transition probability matrix is $1/\mathcal{Q}$. BPSK modulation is employed, and hence $S = 2$, therefore $[\Pi_0]_{q,q'} = \frac{1}{\mathcal{Q}} = 0.25$ where $\mathcal{Q} = S^{N_T}$. BPSK signals are obtained by generating binary random i.i.d. signals with uniform distribution.

TABLE III

IMM-SIC ALGORITHM FOR MIMO STC SYSTEM.

IMM-SIC is initialized at i=0 with $P(m^0_{q_\kappa}|\mathbf{Z}^0_{1:\kappa}) = \frac{1}{\mathcal{Q}}$ and $\mathbf{c}_{q,\kappa}[0] \sim \mathcal{N}(\bar{\mathbf{c}}_0, \bar{\mathbf{\Sigma}}_0)$; For sequence length $i = 1, \ldots, M$ the IMM-SIC is called recursively.

FOR $ti = 2i : 2i+1$ DO

FOR $\kappa = 1, \ldots, K$ DO

1. All Kalman filters are initialised with mixed channel estimates. Mixing probabilities (weights) $\mu_{q|q'}[ti]$ are given by, $P(m^{ti-1}_{q'_\kappa}|m^{ti}_{q_\kappa}, \mathbf{Z}^{ti-1}_{1:\kappa}) = \frac{P(m^{ti-1}_{q'_\kappa}|m^{ti}_{q_\kappa})P(m^{ti-1}_{q'_\kappa}|\mathbf{Z}^{ti-1}_{1:\kappa})}{\sum_{q'_\kappa=1}^{\mathcal{Q}} P(m^{ti-1}_{q'_\kappa}|m^{ti}_{q_\kappa})P(m^{ti-1}_{q'_\kappa}|\mathbf{Z}^{ti-1}_{1:\kappa})}$ used in calculating the mixed estimates (or prior distributions), $p(\mathbf{c}_\kappa[ti]|m^{ti}_{q_\kappa}, \mathbf{Z}^{ti-1}_{1:\kappa}) \sim \mathcal{N}(\hat{\mathbf{c}}^0_{q,\kappa}[ti], \mathbf{\Sigma}^0_{q,\kappa}[ti])$ for $\forall q_\kappa$. $P(m^{ti-1}_{q'_\kappa}|m^{ti}_{q_\kappa}) = \pi_{qq'}$ where $\pi_{qq'} = [\Pi_0]_{qq'}$ or $[\Pi_1]_{qq'}$ on $ti = 2i$ or $2i+1$ respectively.

FOR $q_\kappa = 1, \ldots, \mathcal{Q}$ DO

2. Compute one-step predictive update for q_κth Kalman filter,
$\hat{\mathbf{c}}_{q,\kappa}[ti|ti-1] = \mathbf{F}_\kappa \hat{\mathbf{c}}^0_{q,\kappa}[ti]; \quad \mathbf{\Sigma}_{q,\kappa}[ti|ti-1] = \mathbf{H}_{q,\kappa}[ti]\mathbf{\Sigma}^0_{q,\kappa}[ti]\mathbf{H}_{q,\kappa}[ti]^\top + \rho_w^2 \mathbf{I}_{N_T N_R}$
3. Compute innovation likelihood for user κ, $p(\mathbf{z}_\kappa[ti]|m^{ti}_{q_\kappa}, \mathbf{Z}^{ti-1}_{1:\kappa}, \mathbf{z}_{1:\kappa-1}[ti]) \sim \mathcal{N}(\epsilon_{q,\kappa}, \mathbf{S}_{q,\kappa})$ with $\epsilon_{q,\kappa}$, $\mathbf{S}_{\mathbf{q},\kappa}$
This step is very similar in spirit to the standard successive cancellation [41]. Note that in IMM-SIC not only the MUI estimate term is cancelled via $\bar{\mathbf{V}}_\kappa[ti]$ but its uncertainty is also considered via $\mathrm{Var}\{\mathbf{V}_\kappa[ti]|\mathbf{Z}^{ti-1}_{1:\kappa}, \mathbf{z}_{1:\kappa-1}[ti]\}$; essentially a soft cancellation procedure (55),(56).
4. Compute one-step Kalman filtering update, $\hat{\mathbf{c}}_{q,\kappa}[ti|ti] = \mathbf{E}\{\mathbf{c}_{q,\kappa}[ti]|m^{ti}_{q_\kappa}, \mathbf{Z}^{ti}_{1:\kappa}\}$;
$\mathbf{\Sigma}_{q,\kappa}[ti|ti] = \mathrm{Cov}\{\mathbf{c}_{q,\kappa}[ti]|m^{ti}_{q_\kappa}, \mathbf{Z}^{ti}_{1:\kappa}\}$.

END

5. Compute all the mode probabilities;
$\mu_{q,\kappa}[ti] = P(m^{ti}_{q_\kappa}|\mathbf{Z}^{ti}_{1:\kappa}) = \frac{1}{a} p(\mathbf{z}_\kappa[ti]|m^{ti}_{q_\kappa}, \mathbf{Z}^{ti-1}_{1:\kappa}, \mathbf{z}_{1:\kappa-1}[ti]) \sum_{q'_\kappa=1}^{\mathcal{Q}} P(m^{ti-1}_{q'_\kappa}|m^{ti}_{q_\kappa})P(m^{ti-1}_{q'_\kappa}|\mathbf{Z}^{ti-1}_{1:\kappa})$.
6. MAP estimation; $q^{MAP}_\kappa = \max_q \mu_{q,\kappa}[ti]$ and $\hat{\mathbf{B}}_\kappa[ti] = \bar{\mathbf{b}}_{q^{MAP}_\kappa} \otimes \mathbf{I}_{N_R}$
7. Compute $\mathbf{V}_{\kappa+1}[ti]$ as in (56) using the estimates $\{\hat{\mathbf{B}}_\kappa[ti], \hat{\mathbf{c}}_{q^{MAP},\kappa}[ti|ti], \mathbf{\Sigma}_{q^{MAP},\kappa}[ti|ti]\}$ associated with the MAP mode, $m_{q^{MAP}_\kappa}$, i.e., $\mathbf{V}_{\kappa+1}[ti] \sim \mathcal{N}(\bar{\mathbf{V}}_{\kappa+1}[ti], \mathrm{Var}\{\mathbf{V}_{\kappa+1}[ti]|\mathbf{Z}^{ti-1}_{1:\kappa+1}, \mathbf{z}_{1:\kappa}[ti]\})$, $[\hat{b}_{\kappa,1}[ti], \hat{b}_{\kappa,2}[ti]] = \bar{\mathbf{b}}_{q^{MAP}_\kappa}$.
8. IF mod(ti,2) == 1,

$$\text{Symbol detection} : [\hat{\beta}_{\kappa,0}[i], \hat{\beta}_{\kappa,1}[i]] = \bar{\mathbf{b}}^*_{q^{MAP}_\kappa} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

END

END

END

The complex channel gains are generated by passing two zero mean Gaussian signals through a 3-rd order Butterworth filter with fading rate $f_D T$ (cutoff frequency of the filter corresponding to normalized Doppler frequency) to construct a signal model with complex, rapidly time-varying channel gains. The complex and real components of additive Gaussian noise is generated by two normal distributions of zero-mean and variance ρ^2, therefore the noise variance $\rho_v^2 = 2\rho^2$. We assume transmitted signal powers $A_1^1 = \ldots = A_K^{N_T} = A = 5$ and are known at the receiver.

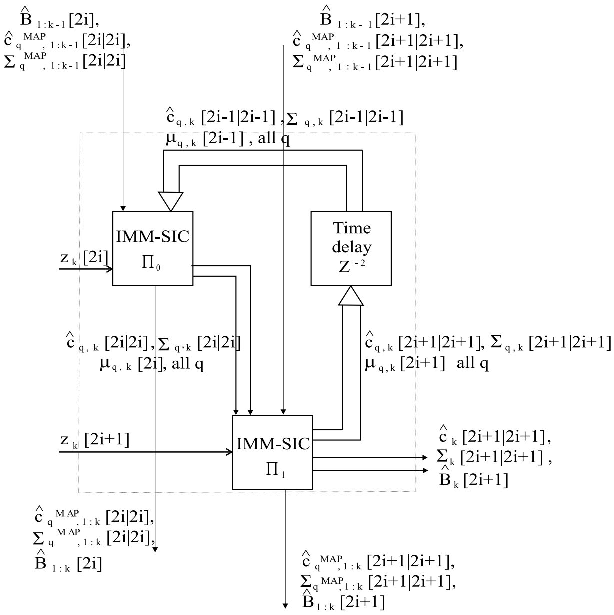
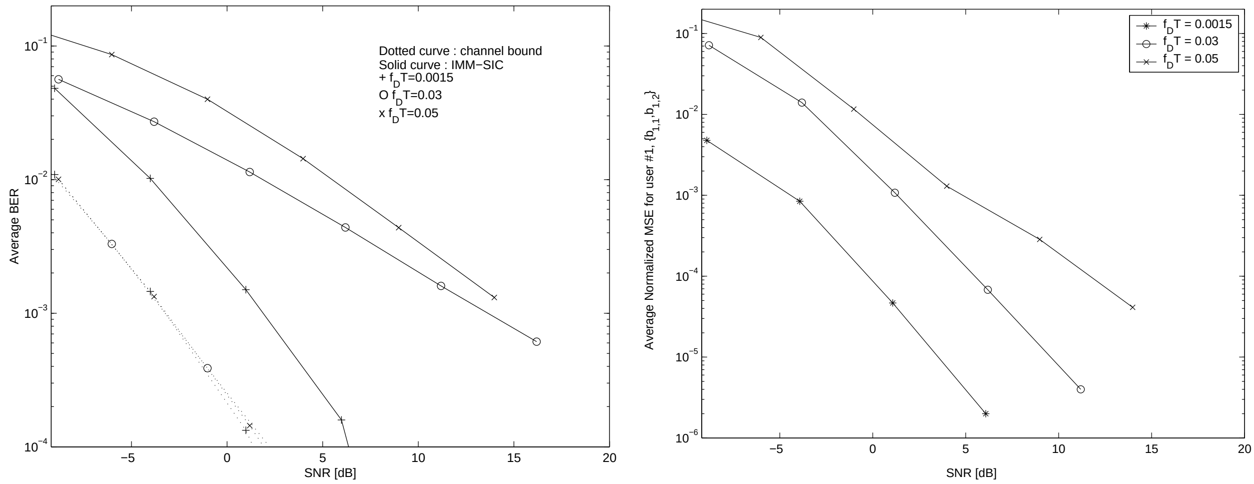


Fig. 4. Diagrammatic representation of IMM-SIC for MIMO STC system at time $2i$ and $2i+1$.

BER performance was measured via 20 Monte-Carlo simulations of sequence length $T = 50000$. The BER performance for increasing fading rates was compared. Fading parameters considered were $f_D T = 0.0015, 0.03, 0.05$, second and third fading rates are fast fading scenarios. Driving nose variance is a design parameter and values which gave reasonable BER performance for above fading rates was chosen, i.e., $\rho_w^2 = 10^{-5}, 4\times10^{-3}, 6\times10^{-2}$ respectively. Measurement noise variance ρ_v^2 can typically be measured at the receiver and is varied in the simulations for different SNR[1] conditions. Phase ambiguity is inherent in joint channel tracking and symbol detection and pilot symbols were used to resolve this ambiguity. Most importantly, insertion of pilot symbols is only to effectively resolve the phase ambiguity, and not due to the need of effectively obtaining the fading channel gain. The data sequence for each user is segmented in to blocks of length $(N_p + N_i)N_T$ symbols. At the beginning of each segment the channels are estimated based on the $N_p N_T$ pilot symbols and then the $N_i N_T$ symbols are detected while simultaneously continuing to estimate the channels $\{c_{k,l}^{n_t,n_r}\}_{n_t=1,n_r=1}^{N_T,N_R}$. For faster fading rates, the spacing of the pilot sequences needs to be denser in order to keep the performance close to the known channel bounds. Therefore for $f_D T = 0.0015, 0.03, 0.05$, $N_p = 10$ and $N_i = 50, 20, 10$ respectively. As seen by the results in Fig. 5(a) IMM-SIC detector operates in low SNR region of < 15dB with good BER performance for fast fading channels. When pilot symbols are transmitted, only the mode probability and state vector

[1]Signal-to-Noise Ratio (SNR) $= A^2 \frac{\mathbf{E}\{\mathbf{c}_k[i]^H \mathbf{c}_k[i]\}}{\rho_v^2}$; $\mathbf{E}\{\}$ is the expectation operator.



(a) Average BER of $\{b_{k,n_t}\}$ vs. SNR.

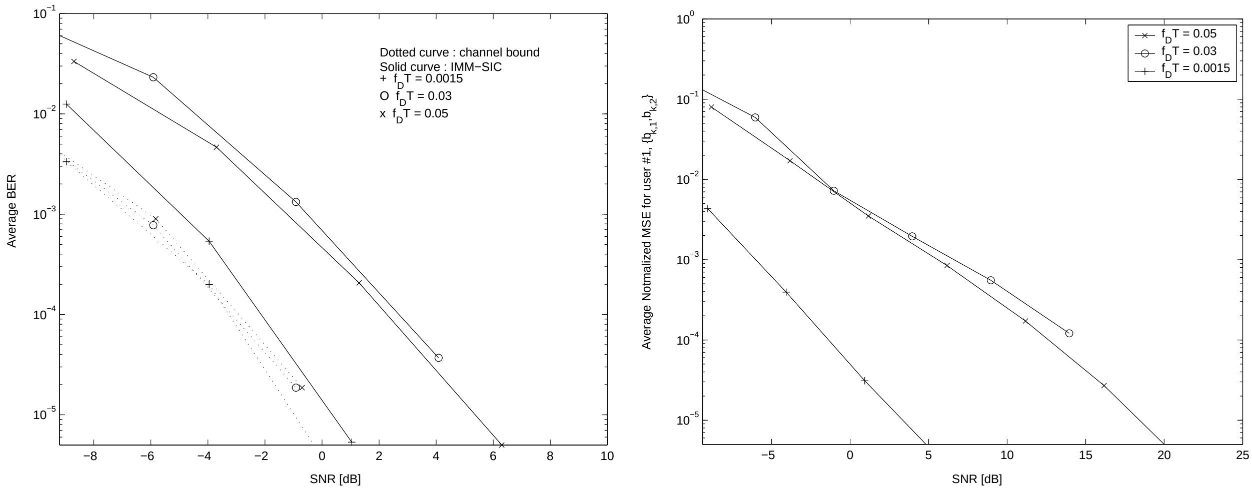
(b) Averaged Normalized MSE for user #1.

Fig. 5. IMM-SIC for MIMO system with $K = 3, L = 1, N_T = 2$.

associated with the pilot symbols have a nonzero probability. Thus the phase ambiguity is resolved at that time since the competing hypotheses with incorrect phase are eliminated. At high SNR and low fading rate the incorrect hypotheses have less effect and therefore phase certainty lasts longer. Differential encoding is another option available for resolving phase ambiguity resulting in a blind implementation, i.e. pilot symbols are not needed. The two bit streams of each user, $\{b_{k,1}, b_{k,2}\}$ are decoded simultaneously at each time-instant. Channel is tracked using a Random-Walk channel model with disturbance variance ρ_w^2. This value was chosen by trial and error based on which value gave the best tracking performance and Fig. 5(b) shows the normalized Mean-square-error (MSE)[2] for different fading rates. The results show that the IMM-SIC is an online algorithm which tracks the varying characteristics of the channel and is well suited for channel estimation, and simultaneously detecting the symbols transmitted, in fast fading scenarios of $f_D T = 0.03, 0.05$.

Next example is for a MIMO system with multipath where $\Upsilon_{k,1} = 0$, $\Upsilon_{k,2} = 1$, $\Upsilon_{k,3} = 2$. As shown in Fig. 6 (a), in this case IMM-SIC has improved performance over a system with no multipath. This is expected as multipath provides more information regarding the transmitted symbol sequence but MSE performance decreases due to multipath fading, shown in Fig. 6(b), compared to MIMO system with no multipath shown in Fig. 5(b). The design parameter ρ_w^2 is a trade-off between BER performance and channel tracking. The results in Fig. 7(a) is for channel fading of $f_D T = 0.03$ and shows channel tracking capability of IMM-SIC for the chosen value

[2]Average Norm. $\text{MSE}_k = \frac{1}{N_T N_R}\sum_{n_t=1}^{N_T}\sum_{n_r=1}^{N_R}\text{NMSE}_k^{n_t,n_r}$ where $\text{MSE}_{k,l}^{n_t,n_r}[i] = (c_{k,l}^{n_t,n_r}[i] - \hat{c}_{k,l}^{n_t,n_r}[i])$; $\text{NMSE}_k^{n_t,n_r} = \frac{\sum_{l=1}^{L}\mathbf{E}\{\text{MSE}_{k,l}^{n_t,n_r}[i]^H\text{MSE}_{k,l}^{n_t,n_r}[i]\}}{\sum_{l=1}^{L}\mathbf{E}\{c_{k,l}^{n_t,n_r}[i]^H c_{k,l}^{n_t,n_r}[i]\}}$.



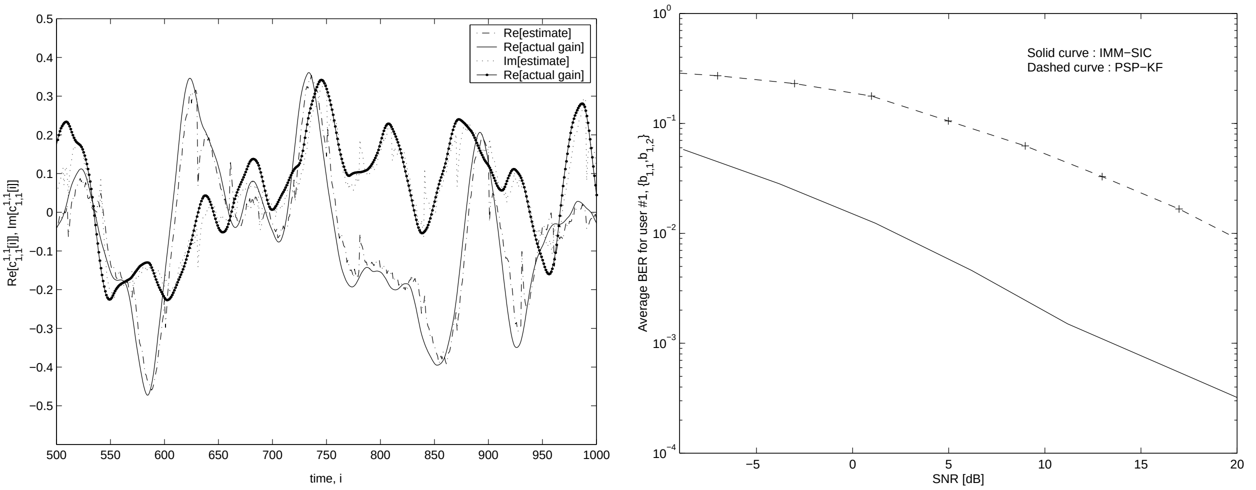
(a) Average BER of $\{b_{k,n_t}\}$ vs. SNR.

(b) Average Normalized MSE for user #1.

Fig. 6. IMM-SIC for MIMO system with $K = 3$, $L = 3$, $N_T = 2$.

$\rho_w^2 = 4 \times 10^{-3}$.

2) MIMO symbol estimation with Per-survivor processing: For performance comparison, we use the PSP detector, a sequence estimation algorithm with a trellis of possible transmitted symbols with a fixed trellis size. Even though the complexity of a PSP algorithm will be prohibitive for medium to large users it is a maximum likelihood (ML) based data detector and we evaluate its performance in comparison to the IMM-SIC algorithm using the same signal model. Actual physical channel memory is unknown and the channel is tracked using the same channel model as for the IMM-SIC algorithm (18). That is each state corresponds to the current symbol set at each time-instant and sequence length considered by the PSP is $L_m = 1$. For a BPSK transmission ($S = 2$) the size of the symbol constellation is 2 but for $N_T = 2$ the constellation size at each time-instant will be $S^{N_T} = 4$. Therefore the total number of states in the trellis for a MIMO system will be $P = 4^{L_m} = 4$. In a purely blind signaling environment it has been demonstrated that there exist equivalent sequences of the symbol data [43], [44] which traditionally necessitates the use of pilot symbols. We adaptively track the channel states using PSP-KF [43], [44] based algorithm (channel tracking is by P parallel Kalman filters operating in parallel). For each state at each time instant there are 4 transitions emerging from it and going to 4 different states: each transition corresponds to one of the 4 possible choices for the symbol pair $[\hat{b}_{k,1}, \hat{b}_{k,2}]$. There is a branch metric associated with each transition and the PSP-KF detector has one survivor per state selected based on maximum branch likelihood metric out of 4 possible transitions. A one-step Viterbi algorithm is then performed to calculate the accumulated metrics of the states at each time-



(a) Channel tracking.

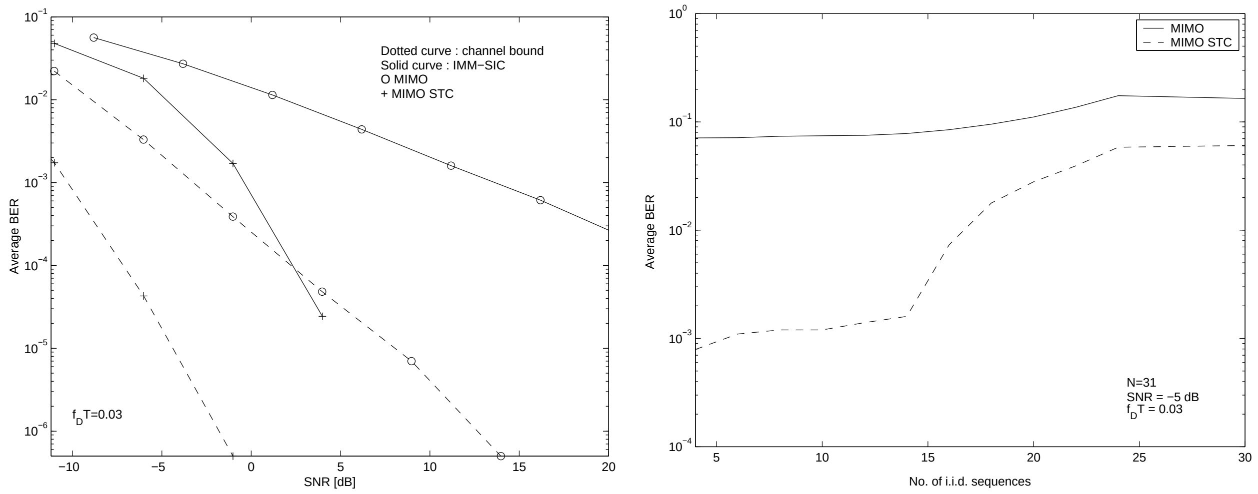
(b) IMM-SIC vs. PSP-KF. PSP is with 4 states ($P = \mathcal{Q}^{L_m}$).

Fig. 7. IMM-SIC and PSP-KF for MIMO system. $f_D T = 0.03$, $K = 3$, $L = 1$, $N_T = 2$, $N_p = 10$, $N_i = 20$.

instant. The measurement equation for the Kalman filter will be (57). The channel estimates $\hat{\mathbf{c}}_k[i|i]$ and $\hat{\mathbf{\Sigma}}_k[i|i]$ at each state are updated by a Kalman filter initialized with the posterior estimates of the survivor state leading to it.

Channel is estimated adaptively during N_i information symbols and N_p pilot symbols. The N_i information bit pairs are detected based on the ML solution of the accumulated metric. Roughly speaking, the best survivor (with the ML solution) is extended back along the sequence length which gives a sequence of surviving states which represent the symbols transmitted by $N_T = 2$ transmitters during each time instant. The BER performance of this estimation algorithm is compared with IMM-SIC detector in Fig. 7(b). Computational complexity at each time instant for the PSP-KF presented here for a single user is $O(Pn^3)$, whereas IMM-SIC for a single user has a complexity of $O(\mathcal{Q}n^3)$ where $P = 4^{L_m}, \mathcal{Q} = 4, n = LN_TN_R$. The PSP-KF can be implemented for $L_m > 1$ but this increases exponentially the number of states in the trellis diagram. For a fast fading channel tracked using a Random-Walk model the IMM-SIC algorithm has better performance than the PSP-KF estimator, as shown in Fig. 7(b).

3) IMM-SIC symbol detector for MIMO STC system: Considering a MIMO system with STC, simulations are carried out for $N_T = 2$, using Alamouti STC. The Markov property of the STC signals determines the transition probability matrix Π_1 with elements of the matrix taking values of either 1 or 0 and the Markov property of the original sequence determines the matrix Π_0. The results are compared against a lower performance bound with known channel information. Each



(a) Average BER vs. SNR, $K = 3$.

(b) Average BER vs. number of i.i.d. sequences.

Fig. 8. Comparison of IMM-SIC for MIMO and MIMO STC systems. $L = 1$, $N_T = 2$.

user's symbol streams $\{b_{k,1}, b_{k,2}\}$ are decoded simultaneously at each time-instant and the average results are shown in Fig. 8(a).

Lastly, the IMM-SIC algorithm is evaluated for increasing number of users $K = 2, \ldots, 15$. The average BER over all KN_T transmission sequences is computed. The load on the system is $\frac{KN_T}{N} \times 100\%$. As seen in Fig. 8(b), performance decreases with increasing load for a fast fading channel at low SNR with maximum possible load being $K = 15 (\approx 96.77\%)$ users for $N = 31$ and $N_T = 2$.

VII. Conclusion

A recursive suboptimal algorithm is derived using Bayesian statistics with approximations made at two levels. Firstly, an approximation that truncates the consideration of the symbol sequence to just the current and past symbol is made, but still requiring all users information to be available together, leading to the interacting multiple model (IMM) algorithm. Secondly, an approximation that considers only the previous users estimated symbol and channel information, similar to the successive interference cancellation (SIC) procedure, is made, leading to the derivation of the IMM-SIC detector. The computational complexity of the algorithm is $O(KQn^3)$, and is linear in number of users and time. Importantly, the algorithm can be applied recursively to space-time (ST) coded transmissions, avoiding the need for processing after receipt of all the ST coded transmitted signals as in most literature on STBC MIMO systems, due to its ability to handle Markov source generated symbol sequences. The paper illustrates how the detector processes the received signals of the Alamouti transmission matrix in a recursive manner.

The results show that detection of Alamouti encoded symbol transmission in time-varying channels is ideal for the IMM-SIC detector, especially for joint channel estimation and symbol detection occurring concurrently with estimates directly influencing each other. The detector (with or without STBC) has only $\mathcal{Q} = S^{N_T}$ system modes at each time-instant and has better performance at reduced complexity compared to the per-survivor sequence detector. IMM-SIC detector with ST decoding had better performance than IMM-SIC leading to the conclusion that the mode transitions based ST decoding is advantageous for systems with ST coded transmissions where the received signals are processed recursively. Despite the reduced computational complexity, IMM-SIC detector has shown significant performance over a wide range of fading rates and noise levels.

VIII. Acknowledgment

The authors thank Manuel A. Sanchez-Lorenzo and the anonymous reviewers for their helpful comments. The financial support from the Automatic Control & Systems Engineering Dept. at Sheffield is acknowledged.

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