Reversed Processes of Multiple Agent Cooperations

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Abstract. The Reversed Compound Agent Theorem (RCAT) is a compositional result that uses Markovian process algebra (MPA) to derive the reversed process of a cooperation between two agents. The equilibrium state probability distribution follows directly from a reversed process, resulting in a product-form solution for the joint state probabilities. This paper generalizes RCAT to multiple (more than two) cooperating agents. This greatly reduces the complexity of applying the original RCAT in multiple agent cooperations, removing the need for multiple applications and inductive proofs. The principle advantage is the potential of mechanically applying Multiple Agents RCAT to complicated networks, such as G-networks and BCMP networks, giving automated proofs of product-forms.

1 Introduction

Stochastic process algebra (SPA) is an extension of classical process algebra with time delays and probabilities, aimed at providing performance descriptions of concurrent systems. The inherent compositional structure separates the model of a system into successively more fundamental components and, through the interactions among the components, performance characteristics of complex systems can be assessed. This facilitates the development of efficient computer and communication systems by providing crucial performance analysis during the design phase.

In the last decade, a number of modelling formalisms in the family of SPA have been developed, such as Timed Processes and Performance Evaluation (TIPP) [9], Performance Evaluation Process Algebra (PEPA) [10], which are both Markovian, and Extended Markovian Process Algebra (EMPA) [2]. TIPP is the first process algebra to be used for performance modelling. PEPA is the smallest language in terms of combinators. EMPA introduced weights for immediate actions. We have choosen part of PEPA for our investigation, using the *prefix* and the *cooperation* combinators.

Product-form queueing networks play a vital role in performance analysis, for example Jackson networks [11], BCMP networks [1], Boucherie networks [3], and G-networks [4]. The core of our research is the Reversed Compound Agent Theorem (RCAT) [5], which exhibits product-form solution through reversed processes using the PEPA formalism. The significant achievement of this methodology is to unify many existing product-forms for diverse networks, including those referred to above. This paper generalizes RCAT to multiple (more than two) cooperative agents. This greatly reduces the complexity of applying the original RCAT in multiple agent cooperations, removing the need for multiple applications and inductive proofs.

2 Multiple Agents RCAT

In the PEPA cooperation $P \underset{L}{\bowtie} Q$, we denote the subset of action types in a cooperation set L which are *passive* with respect to an agent P by $\mathcal{P}_P(L)$ and the subset of corresponding *active* action types by $\mathcal{A}_P(L) = L \backslash \mathcal{P}_P(L)$, similarly for agent Q. In our analysis, we assume that, in $P \underset{L}{\bowtie} Q$, any active action in P has a corresponding passive action in Q, and vice versa; therefore, $\mathcal{P}_P(L) = \mathcal{A}_Q(L)$ and $\mathcal{A}_P(L) = \mathcal{P}_Q(L)$.

Now consider a multiple agent cooperation $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$ with $n \geq 2$, where L_k is the set of cooperation action types that occur in agent P_k, $\mathcal{P}_{P_k}(L_k) \subset \bigcup_{\substack{j=1 \\ j \neq k}}^{n} \mathcal{A}_{P_j}(L_k)$ and $\mathcal{A}_{P_k}(L_k) \subset \bigcup_{\substack{j=1 \\ j \neq k}}^{n} \mathcal{P}_{P_j}(L_k)$ as each of the n agents cooperate with at most one other. More specifically, $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$ is an abbreviation of

$$(\ldots((P_1 \underset{L_2}{\bowtie} P_2) \underset{L_3}{\bowtie} P_3) \underset{L_4}{\bowtie} \ldots) \underset{L_n}{\bowtie} P_n$$

In fact, we can also write this as

$$P_1 \underset{L}{\bowtie} P_2 \underset{L}{\bowtie} P_3 \underset{L}{\bowtie} \ldots \underset{L}{\bowtie} P_n$$

where $L = \bigcup_{k=1}^{n} L_k$. We now define the following notation, similar to [7]:

$\mathcal{P}_{P_k}^{i_k \rightarrow}$ denotes the set of action types in L_k that are *passive* in P_k and correspond to transitions *out of* state i_k in the Markov process of P_k;

$\mathcal{P}_{P_k}^{i_k \leftarrow}$ denotes the set of action types in L_k that are *passive* in P_k and correspond to transitions *into* state i_k in the Markov process of P_k;

$\mathcal{A}_{P_k}^{i_k \rightarrow}$ denotes the set of action types in L_k that are *active* in P_k and correspond to transitions *out of* state i_k in the Markov process of P_k;

$\mathcal{A}_{P_k}^{i_k \leftarrow}$ denotes the set of action types in L_k that are *active* in P_k and correspond to transitions *into* state i_k in the Markov process of P_k;

$\mathcal{P}^{\underline{i}\rightarrow}$ denotes the set of action types in $L = \bigcup_{k=1}^{n} L_k$ that are *passive* and correspond to transitions *out of* state $\underline{i} = (i_1, i_2, \ldots, i_n)$ in the Markov process of $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$;

$\mathcal{P}^{\underline{i}\leftarrow}$ denotes the set of action types in L that are *passive* and correspond to transitions *into* state $\underline{i} = (i_1, i_2, \ldots, i_n)$ in the Markov process of $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$;

$\mathcal{A}^{\underline{i}\rightarrow}$ denotes the set of action types in L that are *active* and correspond to transitions *out of* state $\underline{i} = (i_1, i_2, \ldots, i_n)$ in the Markov process of $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$;

$\mathcal{A}^{\underline{i}\leftarrow}$ denotes the set of action types in L that are *active* and correspond to transitions *into* state $\underline{i} = (i_1, i_2, \ldots, i_n)$ in the Markov process of $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$;

$\alpha_a^{\underline{i}}$ denotes the instantaneous transition rate *out of* state $\underline{i} = (i_1, i_2, \ldots, i_n)$ in the Markov process of $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$ corresponding to *active* action type $a \in L$.

$\overline{\beta_a^{\underline{i}}}$ denotes the instantaneous transition rate *out of* state $\underline{i} = (i_1, i_2, \ldots, i_n)$ in the reversed Markov process of $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$ corresponding to *passive* action type $a \in L$; note that a is *incoming* to state $\underline{i} = (i_1, i_2, \ldots, i_n)$ in the forwards process.

We now in a position to state and prove the main result of this paper, the Multiple Agents RCAT. First we give the rule of [5] about how to apportion the rates in reversed multiple actions.

Definition 1. *The reversed actions of multiple actions* (a_i, λ_i) *for* $1 \leq i \leq m$ *that an agent* $P_k(k = 1, 2, \ldots, n)$ *can perform, are respectively*

$$(\overline{a_i}, (\lambda_i/\lambda)\overline{\lambda})$$

where $\lambda = \lambda_1 + \cdots + \lambda_m$ *and* $\overline{\lambda}$ *is the reversed rate of the one-step, composite transition with rate* λ *in the underlying Markov chain of* $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$*, corresponding to all the combined forward arcs.*

Theorem 1. *Suppose that the cooperation* $\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k$ *has a derivation graph with an irreducible subgraph* G*. Given that every instance of a reversed action of an active action type in* $\mathcal{A}_{P_k}(L_k)$ *has the same rate in* $\overline{P_k}$*, where* $k = 1, 2, \ldots, n$*, the reversed agent* $\overline{\underset{L_k}{\overset{n}{\underset{k=1}{\bowtie}}} P_k}$*, with derivation graph containing the reversed subgraph* $\overline{G}$*, is*

$$\underset{\overline{L_k}}{\overset{n}{\underset{k=1}{\bowtie}}} \overline{R_k}\{(\overline{a}, \overline{p_{k_a}}) \leftarrow (\overline{a}, \top) \,|\, a \in \mathcal{A}_{P_k}(L_k)\}$$

where

$$R_k = P_k\{\top_a \leftarrow x_a \,|\, a \in \mathcal{P}_{P_k}(L_k)\} \qquad k = 1, 2, \ldots, n \tag{1}$$

$\{x_a\}$ are the solutions (for $\{\top_a\}$) of the equations

$$\{\top_a = \overline{p_k}_a \,|\, a \in \mathcal{A}_{P_k}(L_k)\} \qquad k = 1, 2, \ldots, n \tag{2}$$

$\overline{p_k}_a$ is the symbolic rate of action type $\overline{a}$ in $\overline{P_k}$, provided that

$$\sum_{a\in\mathcal{P}^{\underline{i}\rightarrow}} x_a - \sum_{a\in\mathcal{A}^{\underline{i}\leftarrow}} x_a = \sum_{a\in\mathcal{P}^{\underline{i}\leftarrow}\setminus\mathcal{A}^{\underline{i}\leftarrow}} \overline{\beta_a^{\underline{i}}} - \sum_{a\in\mathcal{A}^{\underline{i}\rightarrow}\setminus\mathcal{P}^{\underline{i}\rightarrow}} \alpha_a^{\underline{i}} \tag{3}$$

Proof. Let the instantaneous transition rate in the Markov chain of P_k (respectively R_k) out of state i_k corresponding to action type a be $p_{k;i_k a}$ (respectively $r_{k;i_k a}$) and let $p_{ki_k} = \sum_{a\in\mathcal{O}_{k;i_k}} p_{k;i_k a}$ (similarly for r_{ki_k}), where $\mathcal{O}_{k;i_k}$ is the set of all outgoing action types in the derivative corresponding to state i_k. In other words, p_{ki_k} is the total outgoing rate from state i_k in the Markov chain of P_k.

In $\underset{\substack{k=1\\L_k}}{\overset{n}{\bowtie}} P_k$, the total rate out of any state $\underline{i} = (i_1, i_2, \ldots, i_n) \in G$ is

$$\sum_{k=1}^{n} p_{ki_k}\{\top \leftarrow 0\} - \sum_{k=1}^{n} \sum_{a\in\mathcal{A}_{P_k}^{i_k\rightarrow}\setminus\mathcal{P}^{\underline{i}\rightarrow}} p_{k;i_k a}$$

where $\{\top \leftarrow 0\}$ denotes setting every occurrence of an unspecified rate corresponding to action types in L_k to zero, which is an abbreviation for $\{\top_a \leftarrow 0 \,|\, a \in L_k\}$. Note that $\mathcal{A}_{P_k}^{i_k\rightarrow}\setminus\mathcal{P}^{\underline{i}\rightarrow}$ is the set of active actions in P_k that do not have passive actions to synchronize with in state $\underline{i}$; the disabled active actions do not contribute to the total rate out of state $\underline{i}$. Since $\mathcal{A}_{P_1}^{i_1\rightarrow}, \mathcal{A}_{P_2}^{i_2\rightarrow}, \ldots,$ and $\mathcal{A}_{P_n}^{i_n\rightarrow}$ are disjoint, the total rate out of state $\underline{i}$ in $\underset{\substack{k=1\\L_k}}{\overset{n}{\bowtie}} P_k$ can be simplified to:

$$\sum_{k=1}^{n} p_{ki_k}\{\top \leftarrow 0\} - \sum_{a\in\mathcal{A}^{\underline{i}\rightarrow}\setminus\mathcal{P}^{\underline{i}\rightarrow}} \alpha_a^{\underline{i}}$$

Now, consider the total outgoing rates in the reversed agent $\overline{\underset{\substack{k=1\\L_k}}{\overset{n}{\bowtie}} P_k}$.

$$r_{ki_k} = p_{ki_k}\{\top_a \leftarrow x_a | a \in \mathcal{P}_{P_k}(L_k)\} \qquad k = 1, 2, \ldots, n \tag{4}$$

are the total rates out of state i_k in R_k with $k = 1, 2, \ldots, n$. Thus,

$$r_{ki_k} = p_{ki_k}\{\top \leftarrow 0\} + \sum_{a\in\mathcal{P}_{P_k}^{i_k\rightarrow}} x_a \qquad k = 1, 2, \ldots, n \tag{5}$$

Hence, by the first of Kolmogorov's criteria [12], the total rate out of state $\underline{i} = (i_1, i_2, \ldots, i_n)$ in

$$\mathop{\bowtie}_{\substack{k=1 \\ L_k}}^{n} \overline{R_k}\{(\overline{a}, \overline{p_{ka}}) \leftarrow (\overline{a}, \top) | a \in \mathcal{A}_{P_k}(L_k)\}$$

is

$$\sum_{k=1}^{n} \left(p_{ki_k}\{\top \leftarrow 0\} + \sum_{a \in \mathcal{P}_{P_k}^{i_k \rightarrow}} x_a - \sum_{a \in \mathcal{A}_{P_k}^{i_k \leftarrow}} x_a - \sum_{a \in \mathcal{P}_{P_k}^{i_k \leftarrow} \setminus \mathcal{A}^{\underline{i} \leftarrow}} \overline{p_{k;i_k a}} \right)$$

which can be simplified to:

$$\sum_{k=1}^{n} p_{ki_k}\{\top \leftarrow 0\} + \sum_{a \in \mathcal{P}^{\underline{i} \rightarrow}} x_a - \sum_{a \in \mathcal{A}^{\underline{i} \leftarrow}} x_a - \sum_{a \in \mathcal{P}^{\underline{i} \leftarrow} \setminus \mathcal{A}^{\underline{i} \leftarrow}} \overline{\beta_a^{\underline{i}}}$$

This gives the last condition in the theorem.

According to Kolmogorov's criteria [12], to complete the proof, we need to verify that the product of transition rates in a forward cycle is equal to that in its reversed cycle. Recalling the definition of [5], a base cycle pertains to a cycle with all the arcs originating in only one agent, say P_k. Hence, there are n base cycles for every cycle in $\mathop{\bowtie}_{\substack{k=1 \\ L_k}}^{n} P_k$.

Consider a cycle C with arcs $a_1, a_2, \ldots, a_m$ in $\mathop{\bowtie}_{\substack{k=1 \\ L_k}}^{n} P_k$. Let the corresponding base cycle in P_k have rates $d_{k1}, d_{k2}, \ldots, d_{km}$ where $k = 1, 2, \ldots, n$. Let $D_k = NI_{D_k} \bigcup NN_{D_k} \bigcup A_{D_k} \bigcup P_{D_k}$ be the P_k-base cycle, where NI_{D_k} is the set of arcs of actions in P_k that are not in the cooperation set L_k (but still causing in state changes), NN_{D_k} is the set of arcs of non-cooperative actions in agent P_j for $j \neq k$ (no state changes in P_k), A_{D_k} is the set of arcs of active actions in L_k, and P_{D_k} is the set of arcs of passive actions in L_k.

In fact, all the cycles in a transition graph can be classified into three different kinds; they are composed of arcs in NI_{D_k}, A_{D_k}, or both. Thus, the product of the transition rates around cycle C consists of all the non-cooperating arcs that cause state changes and all the active arcs of each component P_k in $\bigcup_{k=1}^{n} L_k$. However, the transition rate of arc a_i may not be the whole d_{ki}, but a portion of d_{ki} with ratio $\psi_{ii'}$, where i' is the passive arc corresponding to i, $1 \leq i' \leq c_i$, $0 \leq \psi_{ij} \leq 1$, $\sum_{j=1}^{c_i} \psi_{ij} \leq 1$, $\psi_{i0} = 1 - \sum_{j=1}^{c_i} \psi_{ij}$, and c_i is the number of cooperations that the i-th arc in a base cycle that participates as an active action. If an arc a_i refers to a non-cooperating action in P_k, then the transition rate of arc a_i is $\psi_{i0} d_{ki}$. If arc a_i refers to a cooperating action, active in P_k, then the transition rate of arc a_i is $\psi_{ii'} d_{ki}$. Hence, the product of the transition rates around the cycle C is

$$\prod_{k=1}^{n} \left(\prod_{i \in NI_{D_k}} \psi_{i0} d_{ki} \prod_{i \in A_{D_k}} \psi_{ii'} d_{ki} \right)$$

Since every active-passive pair is a 1–1 correspondence, we can obtain all the active arcs in $\bigcup_{k=1}^{n} L_k$ indirectly from all the corresponding passive arcs in $\bigcup_{k=1}^{n} L_k$. Therefore, the product of the transition rates around the cycle C can also be written as:

$$\prod_{k=1}^{n}\left(\prod_{i\in NI_{D_k}} \psi_{i0} d_{ki} \cdot \prod_{l=1}^{n} \prod_{\substack{i'\in P_{D_k}\\ i\in A_{D_l}}} \psi_{ii'} d_{li}\right)$$

In the latter term $\prod_{l=1}^{n} \prod_{\substack{i'\in P_{D_k}\\ i\in A_{D_l}}} \psi_{ii'} d_{li}$, for each passive arc $i' \in P_{D_k}$, we locate its corresponding active arc $i \in A_{D_l}$ uniquely. Then, from $i \in A_{D_l}$, its transition rate can be written as $\psi_{ii'} d_{li}$.

In R_k, arc $a_i \in D_k$ has rate ${d_k}_i^*$ which is the sum of its original rate in P_k (with passive actions assigned rate 0) and the reversed rates of the active actions in P_l (for $l = 1, 2, \ldots, n$ and $l \neq k$) that are bound to the passive actions of a_i in P_k. By definition 1, we have:

$$ {d_k}_i^* = d_{ki} + \sum_{l=1}^{n} \sum_{j\in\Omega_{c_i}} \frac{\psi_{ji} d_{lj}}{{d_l}_j^*} \overline{{d_l}_j^*} \tag{6}$$

where Ω_{c_i} is the set of arcs that actively cooperate with a_i.

Hence the reversed rates around the cycle C is:

$$\begin{aligned}
&\prod_{k=1}^{n}\left(\prod_{i\in NI_{D_k}} \frac{\psi_{i0} d_{ki}}{{d_k}_i^*}\overline{{d_k}_i^*} \prod_{l=1}^{n} \prod_{\substack{i'\in P_{D_k}\\ i\in A_{D_l}}} \frac{\psi_{ii'} d_{li}\overline{{d_l}_i^*}}{{d_l}_i^* {d_k}_{i'}^*}\overline{{d_k}_{i'}^*}\right)\\
&= \prod_{k=1}^{n}\left[\prod_{i\in NI_{D_k}} \psi_{i0} d_{ki} \prod_{i\in NI_{D_k}} \frac{\overline{{d_k}_i^*}}{{d_k}_i^*} \prod_{l=1}^{n}\left(\prod_{\substack{i'\in P_{D_k}\\ i\in A_{D_l}}} \psi_{ii'} d_{li} \prod_{\substack{i'\in P_{D_k}\\ i\in A_{D_l}}} \frac{\overline{{d_l}_i^*}}{{d_l}_i^*} \prod_{\substack{i'\in P_{D_k}\\ i\in A_{D_l}}} \frac{\overline{{d_k}_{i'}^*}}{{d_k}_{i'}^*}\right)\right]\\
&= \prod_{k=1}^{n}\left(\prod_{i\in NI_{D_k}} \psi_{i0} d_{ki} \cdot \prod_{l=1}^{n} \prod_{\substack{i'\in P_{D_k}\\ i\in A_{D_l}}} \psi_{ii'} d_{li}\right)
\end{aligned} \tag{7}$$

by applying Kolmogorov's Criteria to the cycles D_k in R_k where $k = 1, 2, \ldots, n$. This is equal to the product of rates around the forward cycle C.

□

3 Illustrative Example

Consider a 3-node Jackson network, with respective external arrival rates λ_1, λ_2 and λ_3, service rates μ_1, μ_2 and μ_3, and routing probability p_{ij} from node i to node j ($i, j = 1, 2, 3$), where tasks leave the network from node i with probability $\sum_{j \neq i} 1 - p_{ij}$.

This network can be described by the PEPA expression $\underset{L_k}{\overset{3}{\underset{k=1}{\bowtie}}} P_{k,0}$ where:

$$P_{k,n} = (e_k, \lambda_k) P_{k,n+1} \qquad n \geq 0 \tag{8}$$

$$P_{k,n} = (a_{jk}, \top_{jk}) P_{k,n+1} \qquad n \geq 0, j \neq k \tag{9}$$

$$P_{k,n} = (d_k, (1 - \sum_{j \neq k} p_{kj}) \mu_k) P_{k,n-1} \qquad n > 0 \tag{10}$$

$$P_{k,n} = (a_{kj}, p_{kj} \mu_k) P_{k,n-1} \qquad n > 0 \tag{11}$$

with $k, j = 1, 2, 3$, $L_1 = \{a_{12}, a_{13}, a_{21}, a_{31}\}$, $L_2 = \{a_{12}, a_{21}, a_{23}, a_{32}\}$, and $L_3 = \{a_{13}, a_{23}, a_{31}, a_{32}\}$.

In this example, $\mathcal{P}^{\underline{i}\rightarrow} = \mathcal{A}^{\underline{i}\leftarrow} = \bigcup_{k=1}^{3} L_k$ which satisfies the last condition

$$\sum_{a \in \mathcal{P}^{\underline{i}\rightarrow}} x_a - \sum_{a \in \mathcal{A}^{\underline{i}\leftarrow}} x_a = \sum_{a \in \mathcal{P}^{\underline{i}\leftarrow} \setminus \mathcal{A}^{\underline{i}\leftarrow}} \overline{\beta_a^{\underline{i}}} - \sum_{a \in \mathcal{A}^{\underline{i}\rightarrow} \setminus \mathcal{P}^{\underline{i}\rightarrow}} \alpha_a^{\underline{i}} \tag{12}$$

trivially.

Assuming that every occurrence of a reversed action of an active action type in $\mathcal{A}_{P_k}(L_k)$ has the same rate in $\overline{P_k}$, where $k = 1, 2, 3$, we can apply the Multiple Agents RCAT, and obtain the following equations,

$$\top_{21} = p_{21}(\lambda_2 + \top_{12} + \top_{32}) \tag{13}$$

$$\top_{31} = p_{31}(\lambda_3 + \top_{13} + \top_{23}) \tag{14}$$

$$\top_{12} = p_{12}(\lambda_1 + \top_{21} + \top_{31}) \tag{15}$$

$$\top_{32} = p_{32}(\lambda_3 + \top_{13} + \top_{23}) \tag{16}$$

$$\top_{13} = p_{13}(\lambda_1 + \top_{21} + \top_{31}) \tag{17}$$

$$\top_{23} = p_{23}(\lambda_2 + \top_{12} + \top_{32}) \tag{18}$$

Let v_k ($k = 1, 2, 3$) be the visitation rate of node k, which is the total average traffic entering (and leaving) node k in steady state in unit time. We can then simplify the above equations to:

$$\top_{ij} = p_{ij} v_i \qquad i, j = 1, 2, 3 (i \neq j) \tag{19}$$

These are precisely the traffic equations for the internal flows. Therefore, a unique solution is assured with the set of unknowns $\{\top_{ij}\}$ or $\{v_i\}$. Hence the first condition of theorem 1 is satisfied.

By applying the theorem, we can now easily obtain the reversed PEPA agent of $\underset{L_k}{\overset{3}{\underset{k=1}{\bowtie}}} P_{k,0}$, which is $\underset{\overline{L_k}}{\overset{3}{\underset{k=1}{\bowtie}}} X_{k,0}$ where

$$X_{k,n} = (\overline{e_k}, \frac{\lambda_k}{v_k} \mu_k) X_{k,n-1} \qquad n > 0 \tag{20}$$

$$X_{k,n} = (\overline{a_{jk}}, \frac{\top_{jk}}{v_k}\mu_k)X_{k,n-1} \qquad n > 0, j \neq k \tag{21}$$

$$X_{k,n} = (\overline{d_k}, (1 - \sum_{j \neq k} p_{kj})v_k)X_{k,n+1} \qquad n \geq 0 \tag{22}$$

$$X_{k,n} = (\overline{a_{kj}}, \top)X_{k,n+1} \qquad n \geq 0, j \neq k \tag{23}$$

with $k, j = 1, 2, 3$, $\overline{L_1} = \{\overline{a_{12}}, \overline{a_{13}}, \overline{a_{21}}, \overline{a_{31}}\}$, $\overline{L_2} = \{\overline{a_{12}}, \overline{a_{21}}, \overline{a_{23}}, \overline{a_{32}}\}$, and $\overline{L_3} = \{\overline{a_{13}}, \overline{a_{23}}, \overline{a_{31}}, \overline{a_{32}}\}$. Indeed, the rates for the reversed actions can be easily calculated by definition 1. For example, consider the reversed external arrival at node 1, which has type $\overline{e_1}$. The total departure rate of node 1 is μ_1 and the proportion of e_1 in the forward process is $\frac{\lambda_1}{v_1}$. By definition 1, the rate for the reversed action $\overline{e_1}$ is $\frac{\lambda_1}{v_1}\mu_1$.

Compared to the original RCAT, the Multiple Agents RCAT significantly reduces the effort of finding the reversed agent of cooperations in a network with more than two agents.

4 Conclusion

The Multiple Agents Reversed Compound Agent Theorem (MARCAT) greatly simplifies the use of its predecessor, RCAT, for cooperations of an arbitrary number of agents. In terms of automation, its greatest advantage is that inductive proofs of reversed processes, and hence product-forms, are unnecessary. Our simple 3-node example illustrates how MARCAT deals with a triple cooperation. Clearly the same applies to Jackson networks of any number of nodes and similarly to G-networks. However, with the limitation of the theorem to actions that can cooperate between only two processes at a time, e.g. representing departures from one queue passing to another, networks with triggers cannot be handled directly. The method of [8] can be applied to handle this, of course, but MARCAT would be applied to two agents at a time, which is equivalent to the use of the original RCAT. Ongoing research is looking at the case where one active action can cooperate with several passive actions simultaneously. This would not only allow an alternate approach to triggers but could also account for other types of simultaneous movement of customers in queueing networks, for example.

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