

P2P query reformulation over Both-as-View data transformation rules

Peter McBrien¹ and Alexandra Poulouvasilis²

¹ Dept. of Computing, Imperial College, Univ. of London, pjm@doc.ic.ac.uk

² School of Computer Science and Information Systems, Birkbeck College,
Univ. of London, ap@dcs.bbk.ac.uk

Abstract. The both-as-view (BAV) approach to data integration has the advantage of specifying mappings between schemas in a bidirectional manner, so that once a BAV mapping has been established between two schemas, queries may be exchanged in either direction between the schemas. By defining public schemas shared between peers, this allows peers to exchange queries via a public schema without the requirement for any one peer to hold the public schema data.

In this paper we discuss the reformulation of queries over BAV transformation pathways, and demonstrate the use of this reformulation in two modes of query processing. In the first mode, public schemas are shared between peers and queries posed on the public schema can be reformulated into queries over any data sources that have been mapped to the public schema. In the second, queries are posed on the schema of a data source, and are reformulated into queries on another data source via any public schema to which both data sources have been mapped.

1 Introduction

In [MP03a] we presented the **both-as-view (BAV)** approach to data integration, and compared it with **global-as-view (GAV)** and **local-as-view (LAV)** [Len02]. In BAV, schemas are mapped to each other using a sequence of schema transformations which we term a transformation **pathway**. These pathways are reversible, in that a pathway $S_x \rightarrow S_y$ from a schema S_x to a schema S_y may be used to automatically derive the pathway $S_y \rightarrow S_x$ and *vice versa*. Also, from BAV pathways it is possible to extract GAV, LAV and GLAV mapping rules [JTMP04]. The BAV approach has been implemented as part of the AutoMed data integration at Birkbeck and Imperial Colleges (see <http://www.doc.ic.ac.uk/automed>).

As discussed in [MP02,MP03a], one advantage of BAV is that it readily supports the evolution of global and local schemas, including the addition or removal of local schemas. An evolution of a schema S_x to S'_x is expressed as a pathway $S_x \rightarrow S'_x$, and then pathways of the form $S_x \rightarrow S_y$ may be ‘redirected’ to S'_x by prefixing the reverse of $S_x \rightarrow S'_x$, i.e. $S'_x \rightarrow S_x$ to $S_x \rightarrow S_y$ to derive a pathway $S'_x \rightarrow S_y$. As we discussed in [MP03b], this feature makes BAV well-suited to the needs of **peer-to-peer (P2P)** data integration, where peers may join or leave the network at any time, or may change their schemas or pathways between schemas.

Figure 1 illustrates via an example how centralised and P2P data integration differ in BAV. In Figure 1(a), standard centralised data integration of data sources S_1, S_2, \dots into

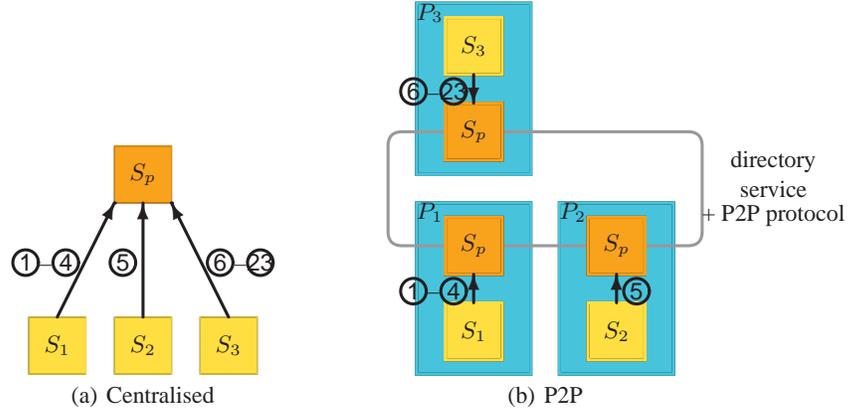


Fig. 1. Example of centralised versus P2P Data Integration in BAV

a global schema S_p is specified by a set of pathways $S_1 \rightarrow S_p, S_2 \rightarrow S_p, \dots$ managed centrally by the data integration system (some of the transformations ①–②③ used to specify these example pathways will be listed later in the paper). In P2P data integration, each peer P_x manages the integration of a data source S_x as a pathway $S_x \rightarrow S_p$, and there is a directory service and P2P protocol that allows the peers to interact¹. The shared global schema is called a **public schema**, emphasising that no single peer controls the global schema but, by contrast, it is simply a publicly available schema definition that any peers may use. Note that the *same* BAV pathway specification is used to map $S_x \rightarrow S_p$ in both the centralised and the P2P systems. The directory service allows a peer to discover what public schemas S_p exist, and which peers support pathways to that public schema. Key aspects of the P2P protocol are [BLMR06]:

- Peer P_x can request that peer P_y executes a query posed on a public schema S_p , and return the results to P_x .
- Peer P_x can request that peer P_y sends it the pathway $S_y \rightarrow S_p$, allowing P_x to itself construct a centralised data integration model and to process queries that access data source S_y without incurring a load on P_y .

One contribution of this paper is that we specify how, given a pathway $S_x \rightarrow S_y$ and a query q posed on S_y , q can be reformulated using a combination of LAV and GAV techniques into a query q' posed on S_x . This is an advance on our previous work which only showed how GAV or LAV views individually could be derived from BAV pathways (we do not consider in this paper reformulation using in addition the GLAV rules that could be extracted from the BAV pathway, and leave that as an area of future work). A second contribution of this paper is that the P2P protocol combined with the reversibility of BAV pathways allows us to support two types of query processing:

- In **public schema querying** we simulate centralised data integration within a P2P environment: a user at a peer P_x poses a query on a public schema S_p , and P_x

¹ For simplicity of presentation in this paper, we assume that each data source is accessed via one peer, and each peer accesses only one data source. In fact, our approach allow a many-many relationship between data sources and peers.

asks each other peer P_y supporting S_p to either (1) process the query and return the result back to P_x , or (2) send its pathway to S_p to P_x so that P_x can construct the centralised data integration model and process the query itself.

- In **data source querying** a user at a peer P_x poses a query q on data source S_x and wishes it to be reformulated into a query q' on some other data source S_y . This is achieved by using the pathway $S_x \rightarrow S_p$ to reformulate q into a query on S_p . Then P_x is able to interact with other peers supporting the public schema S_p , using the public schema querying techniques already described.

Previous work on P2P data integration in the Piazza system has used combinations of LAV and GAV rules between schemas, and a combination of GAV and LAV query processing techniques [HIST03,HIMT03]. Piazza differs from our approach in that mappings must be specified directly between peers. Whilst our approach does not preclude this, we also allow mappings to be specified to a public schema, making our approach more scalable.

Other related work is [Nej03,LNWS03] which uses a superpeer based network topology to provide better scalability than pure peer-to-peer networks. Routing indexes at superpeers store information about the metadata available at the peers directly connected to them, and aid in the forwarding of query requests only to relevant peers.

The need for a superpeer is avoided in the local relational model [BGK⁺02], where peers are directly related by a combination of a domain relation that specifies how the data types of the peers are related, together with coordination formulae that specify that if one predicate is true in one peer, then another predicate is true in another peer.

Our approach combines the respective advantages of these systems by having virtual public schemas — allowing peers to reuse the existing integration of other peers with public schemas — but having no physical superpeer nodes that may act as a bottleneck in the system — in particular, any peer can combine the integrations of other peers with public schemas in order to form direct pathways between peers for query and update processing.

In [CDD⁺03] GLAV rules [FLM99,MH03] are used to specify the constructs of each schema in terms of the constructs of some set of other peer schemas. There is no distinction between source and global schemas, and any number of GLAV rules may be specified between schemas. However, unlike BAV, [CDD⁺03] does not differentiate between sound, complete and exact rules, as the GLAV rules are always sound. CoDB [FKLZ04] generalises this to allow sound and complete GLAV rules to be specified.

The remainder of the paper begins with a review of the BAV data integration approach in Section 2 together with details of the data integration example sketched in Figure 1. We then describe in Section 3 the process of query reformulation over BAV pathways, and illustrate how it supports public schema querying. In Section 4 we discuss how to improve support for data source schema querying, where a certain degree of pathway repair may be needed in order to fully support data source schema querying.

2 Overview of BAV data integration

The basis of the BAV approach to data integration is a low-level **hypergraph-based data model (HDM)**. Higher-level modelling languages are specified in terms of this

lower-level HDM. An HDM schema consists of a set of nodes, edges and constraints, and each modelling construct of a higher-level modelling language is specified as some combination of HDM nodes, edges and constraints. For each type of modelling construct of a modelling language (*e.g.* Table, Column, Primary Key and Foreign Key in the relational model) there are available a set of primitive schema transformations for adding such a construct to a schema, removing such a construct from a schema and, in the case of constructs with textual names, renaming such a construct. Schemas are incrementally transformed by applying to them a sequence of primitive schema transformations, each primitive transformation adding, deleting or renaming just one schema construct.

In general, schema constructs may be extensional *i.e.* have a data extent associated with them (*e.g.* Table and Column in the relational model) or may be constraints (*e.g.* Primary Key and Foreign Key in the relational model). In this paper we will restrict our discussion to the relational model, and hence extensional schema constructs consist of sets of values. The general form of a primitive transformation that adds an extensional construct c of type T to a schema S in order to generate new schema S' is $\text{add}T(c, q_S)$, where q_S is a query over S specifying the extent of c in terms of the existing constructs of S . The semantics of this transformation are that

$$\forall x . x \in c \leftrightarrow x \in q_S$$

In the AutoMed implementation of BAV, q_S is expressed in a functional **intermediate query language (IQL)** (see Section 2.1).

When it is not possible to specify the exact extent of the new construct c being added in terms of the existing schema constructs, the primitive transformation $\text{extend}T(c, \text{Range } q_l q_u)$ must be used instead of add . This adds a new construct c of type T to a schema S , generating a new schema S' . The query q_l over S states what is the minimum extent of c in S' ; q_l may be the constant **Void** if no lower bound on the extent can be specified. The query q_u over S states what is the maximal extent of c in S' , and may be the constant **Any** if no upper bound on the extent can be specified². For non-**Void** q_l therefore, $\forall x . x \in c \leftarrow x \in q_l$; and for non-**Any** q_u , $\forall x . x \in c \rightarrow x \in q_u$. Also, $\text{add}T(c, q_S)$ is equivalent to $\text{extend}T(c, \text{Range } q_S q_S)$

In a similar fashion, the transformation $\text{delete}T(c, q_S)$ when applied to schema S' generates a new schema S with the construct c of type T removed. The extent of c may be recovered using the query q_S on S , and $\forall x . x \in c \leftrightarrow x \in q_S$. Note therefore that from a transformation $\text{delete}T(c, q_S)$ used to transform schema S' to schema S we can automatically infer that $\text{add}T(c, q_S)$ transforms S to S' , and vice versa. When it is not possible to specify the exact extent of the construct c being deleted from S' in terms of the remaining schema constructs, the transformation $\text{contract}T(c, \text{Range } q_l q_u)$ must be used instead of delete . This removes a construct c of type T from schema S' to form a new schema S . The query q_l over S states what is the minimum extent of c in S' , while the query q_u over S states what is the maximal extent of c in S' . Again, q_l may be **Void** and q_u may be **Any**. $\text{delete}T(c, q_S)$ is equivalent to $\text{contract}T(c, \text{Range } q_S q_S)$. Also,

² Syntactically, **Range**, **Void** and **Any** are all examples of **constructors**, which in this case respectively take 2, 0 and 0 arguments. Constructors in functional languages are analogous to function symbols in logic languages.

from $\text{contract}T(c, \text{Range } q_l \ q_u)$ used to transform schema S' to schema S we can infer that $\text{extend}T(c, \text{Range } q_l \ q_u)$ transforms S to S' , and vice versa.

Finally, the transformation $\text{rename}T(c, c')$ causes a construct c of type T in a schema S to be renamed to c' in a new schema S' , where $\forall x . x \in c \leftrightarrow x \in c'$. Thus, from $\text{rename}T(c, c')$ used to transform S to S' we can infer that $\text{rename}T(c', c)$ transforms S' to S .

2.1 AutoMed's IQL Query Language

IQL is a comprehensions-based functional query language³. It supports strings e.g. 'Computer Science', booleans True and False, real numbers, integers, tuples e.g. {1,2,3}, and sets, bags and lists. There are several polymorphic primitive operators for manipulating sets, bags and lists. The operator ++ concatenates two lists, and performs bag union and set union on bags and sets, respectively. The operator flatmap applies a collection-valued function f to each element of a collection and applies ++ to the resulting collections. For sets, it is defined recursively as follows, where [] denotes the empty set and (SCons \times xs) denotes a set containing an element x with xs being the rest of the set (which may be empty):

$\text{flatmap } f \ [] = []$

$\text{flatmap } f \ (\text{SCons } x \ xs) = (f \ x) \ ++ \ (\text{flatmap } f \ xs)$

Henceforth in this paper, we confine our discussion to collections that are sets.

The operator flatmap can be used to specify **comprehensions** over sets. These are of the form $[h \mid q_1; \dots; q_n]$ where h is an expression termed the **head** and q_1, \dots, q_n are **qualifiers**, with $n \geq 0$. Each qualifier is either a **filter** or a **generator**. A generator has syntax $p <- e$ where e is a set-valued expression and p is a **pattern** i.e. an expression involving variables and tuple constructors only. The variables of p are successively bound by iterating through e . Any variables appearing in the head, h , inherit these bindings. A filter is a boolean-valued expression, which must be satisfied by the values generated by the generators in order for these values to contribute to the final result of the comprehension. Comprehensions are a convenient high-level syntax and add no extra expressiveness to languages such as IQL since they translate into applications of flatmap. We give the translation below for a set comprehension, where Q denotes a sequence of qualifiers and $[h]$ a set comprising a single element h :

$[h \mid p <- e; Q] \equiv \text{flatmap } (\lambda p. [h \mid Q]) \ e$

$[h \mid e; Q] \equiv \text{if } e = \text{True} \text{ then } [h \mid Q] \text{ else } []$

$[h \] \equiv [h]$

IQL supports unification of variables appearing in the patterns of generators within the same comprehension. For example,

$[\{a, b, c, d, e\} \mid \{a, b, c\} <- r; \{d, c, e\} <- s]$

is equivalent to $[\{a, b, c, d, e\} \mid \{a, b, c\} <- r; \{d, c2, e\} <- s; c = c2]$

Several equivalences hold for these IQL operators, which follow from their definition and from the interpretation assigned to the Void and Any constants. We list an

³ We refer the reader to [JPZ03] for full details of the language and confine our discussion here to just those aspects that are necessary for this paper. Such languages subsume query languages such as SQL-92 and OQL in expressiveness [Bun94].

indicative subset in the Appendix, including specifically those equivalences that we refer to later in the paper. These equivalences assume that expressions are well-typed (which can be verified statically for IQL) and they are applied by AutoMed’s query optimiser in order to simplify IQL queries before evaluation.

2.2 An Example

Figure 2 shows four schemas S_1, S_2, S_3, S_p . S_1, S_2, S_3 are data source schemas while S_p is what in a centralised data integration system would be called a **global schema** and in our P2P system is called a **public schema**. The semantics of the application domain are that a **student** with name **sname** may repeatedly sit the exam for a **course** (identified by **ccode**, and each having a **title**) over any number of semesters, and achieve an exam **mark** on each exam sitting. However, for all attempts of the course, the student will have the same **tutor** (tutors having been introduced at the start of 1994, along with a coursework mark **cwmark** that students can attempt only once per course). Each student studies for one **degree**. Each degree is identified by a **dcode**, has a title **dname** and has an associated **qualification**.

S_1	studies(sname,ccode,sem,mark,title,dname)		
S_2	teach(sname,ccode,sem,mark,tname?)		
S_3	degree(dcode,dname,qual)	S_p	degree(dcode,dname)
	ug(sname,dcode)		student(sname,dcode)
	reg(sname,ccode,cwmark,tutor)		course(ccode,title)
			sit(sname,ccode,sem,mark,cwmark?)

Fig. 2. Three data sources S_1, S_2, S_3 , and a public schema S_p

Schema S_p is a virtual schema modelling the application domain, omitting the information about tutors and about the qualification associated with degrees. The **cwmark** is shown as optional (by a ‘?’ suffix) since it was only awarded from 1994 onwards. Schema S_1 represents a data source that holds information about courses with a **ccode** greater or equal to 500, and holds data in first normal form (since **dname** is dependent on just **sname** and **title** is dependent on just **ccode**). Schema S_2 represents a data source that holds information about courses with a **ccode** less than 500, and is also in first normal form, since it holds in **tname** the tutor’s name (an optional attribute), which is dependent on just **sname** and **ccode**. Schema S_3 represents a data source that details the tutors of courses, the coursework mark the student gained for the course, and the degrees students studied, and is held in third normal form.

Suppose we wish to transform data or queries between S_p and the other three schemas. We consider below fragments of the pathways $S_1 \rightarrow S_p$ and $S_2 \rightarrow S_p$ in order to illustrate the BAV approach and the use of IQL queries within transformations. Within the pathway $S_1 \rightarrow S_p$ it is necessary to decompose the **studies** table in S_1 in order to produce the separate **course** table that is present in S_p . Here is the fragment of that pathway:

- ① `extendTable(⟨⟨course⟩⟩, Range ([{c} | {s, c, t} <- ⟨⟨studies⟩⟩]) Any)`
- ② `extendColumn(⟨⟨course,ccode⟩⟩, Range [{c, c} | {c} <- ⟨⟨course⟩⟩] Any)`
- ③ `extendColumn(⟨⟨course,title⟩⟩, Range ([{c, ti} | {s, c, t}, ti} <- ⟨⟨studies,title⟩⟩]) Any)`
- ④ `contractColumn(⟨⟨studies,title⟩⟩, Range Void [{s, c, t}, ti} | {s, c, t} <- ⟨⟨studies⟩⟩; {c, ti} <- ⟨⟨course,title⟩⟩])`

Transformation ① states that the `course` table in S_p contains as its set of keys at least those `ccode` attributes of `studies` in S_1 (the first argument of the `Range` constructor). We note here that the AutoMed representation of a relational table models the table itself by its set of primary key values, and models each attribute a of the table by the projection of the table onto the primary key attributes plus a (see [MP03a] for more details of this).

Transformations ② and ③ add the `ccode` and `title` columns to `course`. Again these are `extend` transformations with upper bound `Any`. The final transformation ④ removes the `title` attribute of the `studies` table and specifies the upper bound that the `title` attribute in S_p places on the extent of the `title` attribute in S_1 .

The pathway $S_2 \rightarrow S_p$ needs to specify that the tutor `tname` has no representation in S_p , using transformation ⑤ below. The remainder of the pathway is not required for the examples that follow, and is therefore omitted from our discussion.

- ⑤ `contractColumn(⟨⟨teach,tname⟩⟩, Range Void Any)`

3 Query Reformulation over BAV Pathways

In this section, we discuss how query reformulation can be undertaken over BAV pathways. We first illustrate how BAV pathways can be used for GAV and LAV query reformulation, and hence can support GAV and LAV query processing. We then present a BAV-specific query reformulation algorithm which subsumes as special cases GAV and LAV query reformulation.

3.1 GAV query reformulation

GAV query reformulation is based on query unfolding. For example, to evaluate a query q on S_p with respect to S_1 , we traverse the pathway $S_p \rightarrow S_1$ (i.e. the *reverse* of the pathway $S_1 \rightarrow S_p$ described earlier) replacing each scheme in q that appears in an `delete` or `contract` transformation with the corresponding query of that transformation.

Example Query 1: To reformulate the query

$$q_1 = [{ti} | \{c, ti\} <- \langle \langle \text{course, title} \rangle \rangle; c = 500]$$

first ④ is ignored (since its reverse is an `extend` transformation), and then ③ unfolds $\langle \langle \text{course, title} \rangle \rangle$ giving:

$$[{ti} | \{c, ti\} <- \text{Range}([\{c, ti\} | \{s, c, t, ti\} <- \langle \langle \text{studies, title} \rangle \rangle]) \text{Any}; c = 500]$$

Using the equivalence in Appendix A (a) and the third equivalence in App. A (b) this simplifies to:

$$\text{Range}[\{ti\} | \{c, ti\} <- [\{c, ti\} | \{s, c, t, ti\} <- \langle \langle \text{studies, title} \rangle \rangle]; c = 500] \text{Any}$$

Using the last equivalence in App. A (d) this further simplifies to:

$$\text{Range}[\{ti\} | \{s, c, t, ti\} <- \langle \langle \text{studies, title} \rangle \rangle; c = 500] \text{Any}$$

Transformations ② and ① have no further effect on this query, and thus this is the transformed query that can execute on data source S_1 ⁴.

3.2 Enhanced GAV reformulation

The fact that BAV rules are stated on irreducible constructs of a data model means that GAV query reformulation can be used in some cases where with other data integration approaches LAV query reformulation would have been necessary. To illustrate, consider the pathway $S_3 \rightarrow S_p$, where the table `reg` in S_3 has `sname` and `ccode` as its key attributes since it records the tutor for the course, which does not change for each sitting. This table is mapped to the table `sit` in S_p that has `sname`, `ccode` and `sem` as its key attributes since students may (re)sit the examination part of any course once in any semester. Recall that the tutors for courses were only introduced from `sem 1` of 1994. Below is the relevant fragment of the pathway $S_3 \rightarrow S_p$. We note that transformation ⑥ contains the expression `Const1 s c` in the head of the comprehension. Here, `Const1` is an IQL **constructor** (analogous to a Skolem function in logic languages), used because it is not possible to derive the `sem` attribute of $\langle\langle\text{sit}\rangle\rangle$ from $\langle\langle\text{reg}\rangle\rangle$.

- ⑥ `extendTable($\langle\langle\text{sit}\rangle\rangle$,`
`Range [{s, c, Const1 s c} | {s, c} <- $\langle\langle\text{reg}\rangle\rangle$; (Const1 s c) >= '1994-1'] Any)`
- ⑦ `extendColumn($\langle\langle\text{sit,sname}\rangle\rangle$, Range [{s, c, t}, s] | {s, c, t} <- $\langle\langle\text{sit}\rangle\rangle$] Any)`
- ⑧ `extendColumn($\langle\langle\text{sit,ccode}\rangle\rangle$, Range [{s, c, t}, c] | {s, c, t} <- $\langle\langle\text{sit}\rangle\rangle$] Any)`
- ⑨ `addColumn($\langle\langle\text{sit,cwmark}\rangle\rangle$,`
`[{s, c, t}, cw] | {s, c, t} <- $\langle\langle\text{sit}\rangle\rangle$; [{s, c}, cw] <- $\langle\langle\text{reg,cwmark}\rangle\rangle$)`
- ⑩ `extendColumn($\langle\langle\text{sit,sem}\rangle\rangle$, Range [{s, c, t}, t] | {s, c, t} <- $\langle\langle\text{sit}\rangle\rangle$] Any)`
- ⑪ `deleteColumn($\langle\langle\text{reg,sname}\rangle\rangle$, [{s, c}, s] | {s, c} <- $\langle\langle\text{reg}\rangle\rangle$)`
- ⑫ `deleteColumn($\langle\langle\text{reg,ccode}\rangle\rangle$, [{s, c}, c] | {s, c} <- $\langle\langle\text{reg}\rangle\rangle$)`
- ⑬ `deleteColumn($\langle\langle\text{reg,cwmark}\rangle\rangle$, [{s, c}, cw] | {s, c, t}, cw] <- $\langle\langle\text{sit,cwmark}\rangle\rangle$)`
- ⑭ `contractTable($\langle\langle\text{reg}\rangle\rangle$, Range Void [{s, c} | {s, c, t} <- $\langle\langle\text{sit}\rangle\rangle$; t >= '1994-1'])`

There are a family of constructors `Const1`, `Const2`, ... Any expression of the form `Consti e1 ... en` is only comparable with an expression constructed using the same constructor i.e. with an expression of the form `Consti e'1 ... e'n`. Thus, an expression of the form `Consti e1 ... en = Consti e'1 ... e'n` evaluates to `True` if `ej = e'j` evaluates to `True` for all `j` otherwise it evaluates to `False`, and similarly for the other comparison operators. Any other kind of comparison of `Consti` returns the value `Null`, denoting "unknown". If `Null` is the value of a filter in a comprehension, then the result will be a `Range` expression i.e. the second rule of comprehension translation in Section 2.1 becomes:

$$[h \mid e; Q] \equiv \text{if } e = \text{True} \text{ then } [h \mid Q] \text{ elseif } e = \text{False} \text{ then } [] \text{ else } (\text{Range Void } [h \mid Q])$$

Example Query 2: Consider the following query posed on S_p :

$$q_2 = [{s, c, cw} \mid \{s, c, t}, cw] <- \langle\langle\text{sit,cwmark}\rangle\rangle; t >= '1997-1']$$

Unfolding $\langle\langle\text{sit,cwmark}\rangle\rangle$ using ⑨ we obtain:

$$[s, c, cw] \mid \{s, c, t, cw] <- [\{s, c, t}, cw] \mid \{s, c, t} <- \langle\langle\text{sit}\rangle\rangle; \\ \{s, c}, cw] <- \langle\langle\text{reg, cwmark}\rangle\rangle; t >= '1997-1']$$

⁴ Applications of the simplifications from Appendix A have no effect on the semantics of queries, and we have used them here to improve the readability of our example queries. In practice, the AutoMed logical optimiser applies these kinds of simplifications just once, after the query has been fully reformulated.

which by an equivalence in App. A (d) simplifies to

$$\{\{s, c, cw\} \mid \{s, c, t\} <- \langle\langle sit \rangle\rangle; \{s, c\}, cw <- \langle\langle reg, cwmark \rangle\rangle; t \geq '1997-1'\}$$

Unfolding $\langle\langle sit \rangle\rangle$ using ⑥ we obtain:

$$\{\{s, c, cw\} \mid \{s, c, t\} <- \text{Range}[\{s, c, \text{Const1 } s \ c\} \mid \{s, c\} <- \langle\langle reg \rangle\rangle; (\text{Const1 } s \ c) \geq '1994-1'] \text{ Any}; \{s, c\}, cw <- \langle\langle reg, cwmark \rangle\rangle; t \geq '1997-1'\}$$

By the equivalences of App A (a) and (b), this simplifies to:

$$\text{Range}[\{s, c, cw\} \mid \{s, c, t\} <- [\{s, c, \text{Const1 } s \ c\} \mid \{s, c\} <- \langle\langle reg \rangle\rangle; (\text{Const1 } s \ c) \geq '1994-1']; \{s, c\}, cw <- \langle\langle reg, cwmark \rangle\rangle; t \geq '1997-1'] \text{ Any}$$

Swapping the last two qualifiers of the outer comprehension, and moving $t \geq '1997-1'$ into the inner comprehension (by equivalences in App A (d)) gives:

$$\text{Range}[\{s, c, cw\} \mid \{s, c, t\} <- [\{s, c, \text{Const1 } s \ c\} \mid \{s, c\} <- \langle\langle reg \rangle\rangle; (\text{Const1 } s \ c) \geq '1994-1'; (\text{Const1 } s \ c) \geq '1997-1']; \{\{s, c\}, cw\} <- \langle\langle reg, cwmark \rangle\rangle] \text{ Any}$$

which (by an equivalence in App A (d)) simplifies to

$$\text{Range}[\{s, c, cw\} \mid \{s, c, t\} <- [\{s, c, \text{Const1 } s \ c\} \mid \{s, c\} <- \langle\langle reg \rangle\rangle; (\text{Const1 } s \ c) \geq '1997-1']; \{\{s, c\}, cw\} <- \langle\langle reg, cwmark \rangle\rangle] \text{ Any}$$

At run time this gives the same result as the following query, since $\text{Const1 } s \ c \geq '1997-1'$ evaluates to Null:

$$\text{Range Void } [\{s, c, cw\} \mid \{s, c, t\} <- [\{s, c, \text{Const1 } s \ c\} \mid \{s, c\} <- \langle\langle reg \rangle\rangle]; \{\{s, c\}, cw\} <- \langle\langle reg, cwmark \rangle\rangle]$$

i.e. it returns as an upper bound the student names, courses they have taken and course-work marks obtained from S_3 .

3.3 LAV query processing

The schema S_1 contains a $\langle\langle studies, dname \rangle\rangle$ attribute, that corresponds in S_p to some instances of the join between $\langle\langle student, dcode \rangle\rangle$ and $\langle\langle degree, dname \rangle\rangle$. This is expressed in BAV by the following fragment of the pathway $S_1 \rightarrow S_p$:

- ⑮ extendTable($\langle\langle student \rangle\rangle$, Range $[\{s\} \mid \{s, c, t\} <- \langle\langle studies \rangle\rangle] \text{ Any}$)
- ⑯ addColumn($\langle\langle student, sname \rangle\rangle$, $[\{s, s\} \mid \{s\} <- \langle\langle student \rangle\rangle]$)
- ⑰ extendColumn($\langle\langle student, dcode \rangle\rangle$, Range Void Any)
- ⑱ extendTable($\langle\langle degree \rangle\rangle$, Range $[\{d\} \mid \{s, d\} <- \langle\langle student, dcode \rangle\rangle] \text{ Any}$)
- ⑲ addColumn($\langle\langle degree, dcode \rangle\rangle$, $[\{d, d\} \mid \{d\} <- \langle\langle degree \rangle\rangle]$)
- ⑳ extendColumn($\langle\langle degree, dname \rangle\rangle$, Range $[\{d, dn\} \mid \{s, d\} <- \langle\langle student, dcode \rangle\rangle; \{\{s, c, t\}, dn\} <- \langle\langle studies, dname \rangle\rangle] \text{ Any}$)
- ㉑ contractColumn($\langle\langle studies, dname \rangle\rangle$, Range Void $[\{s, dn\} \mid \{s, d\} <- \langle\langle student, dcode \rangle\rangle; \{d, dn\} <- \langle\langle degree, dname \rangle\rangle]$)

Example Query 3: Consider the following query on S_p :

$$q_3 = [\{s\} \mid \{s, d\} <- \langle\langle student, dcode \rangle\rangle; \{d, dn\} <- \langle\langle degree, dname \rangle\rangle; dn = 'CS']$$

Using GAV, $\langle\langle degree, dname \rangle\rangle$ would unfold using ⑳ and $\langle\langle student, dcode \rangle\rangle$ would then unfold using ⑰, obtaining:

$$\begin{aligned} & \{\{s\} \mid \{s, d\} \leftarrow \text{Range Void Any}; \\ & \quad \{d, dn\} \leftarrow \text{Range}[\{d, dn\} \mid \{s, d\} \leftarrow \text{Range Void Any}; \\ & \quad \{s, c, t\}, dn\} \leftarrow \langle\langle \text{studies}, dname \rangle\rangle \text{ Any}; dn = \text{'CS'} \end{aligned}$$

which simplifies to just Range Void Any, i.e. giving no answers.

However the query q_3 on S_p can yield answers using LAV query processing. There are two main techniques for LAV query processing, the **inverse rule** algorithm [Qia96,DG97] and the **bucket** algorithm [LRO96]. For simplicity we focus here on the former. Using the inverse rule approach, the definition of a construct c by a query of the form $[h \mid Q]$ is inverted in a two-step process. First, replace each variable in Q that does not appear in h by a distinct $\text{Const}i$ with arguments the variable(s) in h . For example, (15) has two such variables, c and t which are replaced by $\text{Const}2 s$ and $\text{Const}3 s$ respectively; while in (21), there is one such variable d , which is replaced by $\text{Const}8 s dn$ (see below). Next, for each generator $p \leftarrow cs$ in Q , generate a query defining cs in terms of $[p \mid h \leftarrow c; Q']$ where Q' consists of all the filters from Q . To illustrate, we list below all the inverse rules derived from the fragment (15)–(21) of the BAV pathway $S_1 \rightarrow S_p$.

$$\begin{aligned} (15.1) \quad & \langle\langle \text{studies} \rangle\rangle = \text{Range Void} [\{s, \text{Const}2 s, \text{Const}3 s\} \mid \{s\} \leftarrow \langle\langle \text{student} \rangle\rangle] \\ (16.1) \quad & \langle\langle \text{student} \rangle\rangle = [\{s\} \mid \{s, s\} \leftarrow \langle\langle \text{student}, sname \rangle\rangle] \\ (18.1) \quad & \langle\langle \text{student}, dcode \rangle\rangle = \text{Range Void} [\{\text{Const}4 s, d\} \mid \{d, d\} \leftarrow \langle\langle \text{degree}, dcode \rangle\rangle] \\ (19.1) \quad & \langle\langle \text{degree} \rangle\rangle = [\{d\} \mid \{d, d\} \leftarrow \langle\langle \text{degree}, dcode \rangle\rangle] \\ (20.1) \quad & \langle\langle \text{student}, dcode \rangle\rangle = \text{Range Void} [\{\text{Const}5 d dn, d\} \mid \{d, dn\} \leftarrow \langle\langle \text{degree}, dname \rangle\rangle] \\ (20.2) \quad & \langle\langle \text{studies}, dname \rangle\rangle = \text{Range Void} [\{\{\text{Const}5 d dn, \text{Const}6 d dn, \text{Const}7 d dn\}, dn\} \mid \\ & \quad \{d, dn\} \leftarrow \langle\langle \text{degree}, dname \rangle\rangle] \\ (21.1) \quad & \langle\langle \text{student}, dcode \rangle\rangle = \text{Range} [\{s, \text{Const}8 s dn\} \mid \{s, dn\} \leftarrow \langle\langle \text{studies}, dname \rangle\rangle] \text{ Any} \\ (21.2) \quad & \langle\langle \text{degree}, dname \rangle\rangle = \text{Range} [\{\text{Const}8 s dn, dn\} \mid \{s, dn\} \leftarrow \langle\langle \text{studies}, dname \rangle\rangle] \text{ Any} \end{aligned}$$

Query processing that requires to use a particular construct can combine the direct definition of the construct within the BAV pathway with all the inverse rules for that construct derived from the BAV pathway. This set of definitions can be combined using a merge function defined as follows, where union and intersect are set union and set intersection:

$$\text{merge} (\text{Range } e1 \ e2) (\text{Range } e1' \ e2') = \text{Range} (\text{union } e1 \ e1') (\text{intersect } e2 \ e2')$$

Returning to our example, when a query is submitted to S_p and answers are required from S_1 , the rules (15), (16), (17), (18), (19), (20), (21.1), (21.2) can be used. In particular, for processing query q_3 above, we have:

$$\langle\langle \text{student}, dcode \rangle\rangle = \text{merge} (17) (21.1) = (21.1) \text{ and } \langle\langle \text{degree}, dname \rangle\rangle = \text{merge} (20) (21.2) = (21.2)$$

Substitution now for $\langle\langle \text{student}, dcode \rangle\rangle$ and $\langle\langle \text{degree}, dname \rangle\rangle$ in q_3 gives:

$$\begin{aligned} & [\{s\} \mid \{s, d\} \leftarrow \text{Range} [\{s, \text{Const}8 s dn\} \mid \{s, dn\} \leftarrow \langle\langle \text{studies}, dname \rangle\rangle] \text{ Any}; \\ & \quad \{d, dn\} \leftarrow \text{Range} [\{\text{Const}8 s dn, dn\} \mid \{s, dn\} \leftarrow \langle\langle \text{studies}, dname \rangle\rangle] \text{ Any}; \\ & \quad dn = \text{'CS'}] \end{aligned}$$

which simplifies to:

$$\begin{aligned} & \text{Range} [\{s\} \mid \{s, d\} \leftarrow [\{s, \text{Const}8 s dn\} \mid \{s, dn\} \leftarrow \langle\langle \text{studies}, dname \rangle\rangle]; \\ & \quad \{d, dn\} \leftarrow [\{\text{Const}8 s dn, dn\} \mid \{s, dn\} \leftarrow \langle\langle \text{studies}, dname \rangle\rangle]; dn = \text{'CS'}] \text{ Any} \end{aligned}$$

which when evaluated would give the same set of answers as:

$$\text{Range} [\{s\} \mid \{s, d\} \leftarrow \langle\langle \text{studies}, dname \rangle\rangle; dn = \text{'CS'}] \text{ Any}$$

3.4 P2P Query Reformulation on BAV pathways

Following the examples presented above, we now summarise how combined GAV and LAV query reformulation can be carried out over a BAV pathway $S_x \rightarrow S_y$, with the aim of obtaining the maximal information that would be derivable from the BAV pathway by means of GAV and LAV query processing techniques.

Suppose we wish to reformulate a query q posed on S_x to be posed with respect to S_y . (We note that, due to the reversibility of BAV pathways, from a pathway $S_x \rightarrow S_y$ it is also possible to reformulate a query q posed on S_y to be posed with respect to S_x . The process is exactly as described below except that now it is with respect to the, automatically derivable, *reverse* pathway $S_y \rightarrow S_x$. This was the scenario illustrated in the examples above, where pathways $S_x \rightarrow S_p$ were used to reformulate queries on S_p so that they could be evaluated on S_x .)

The first step is to construct a set of view definitions, \mathcal{V} , defining constructs in S_x in terms of constructs in S_y . This is undertaken by traversing the pathway $S_x \rightarrow S_y$, and at each transformation step t taking one of the following actions:

- if t is of the form `rename(c, c')` the rule $c = c'$ is added to \mathcal{V} ;
- if t is of the form `delete(c, q)` or `contract(c, q)`, the rule $c = q$ is added to \mathcal{V} ;
- if t is of the form `add(c, q)`, where q is a comprehension referencing schema constructs c_1, \dots, c_n in its generators, then invert the rule $c = q$ (as described in Section 3.3) to obtain a set of rules of the form $c_i = q_i$ for $1 \leq i \leq n$ such that the only scheme referenced in each q_i is c ; add these rules to \mathcal{V} ;
- if t is of the form `extend($c, \text{Range Void } q_u$)`, where q_u is a comprehension as in the case of `add(c, q)`, then invert the rule $c = \text{Range Void } q_u$ to obtain a set of rules of the form $c_i = \text{Range } q_i \text{ Any}$; add these rules to \mathcal{V} ;
- if t is of the form `extend($c, \text{Range } q_l \text{ Any}$)`, where q_l is a comprehension as in the case of `add(c, q)`, then invert the rule $c = \text{Range } q_l \text{ Any}$ to obtain a set of rules of the form $c_j = \text{Range Void } q_j$; add these rules to \mathcal{V} ;
- if t is of the form `extend($c, \text{Range } q_l \text{ } q_u$)`, where q_l and q_u are comprehensions as in the case of `add(c, q)`, then invert the rule $c = \text{Range } q_l \text{ } q_u$ by inverting separately q_u and q_l , as in the previous two cases, to obtain from q_u a set of rules of the form $c_i = \text{Range } q_i \text{ Any}$ and from q_l a set of rules of the form $c_j = \text{Range Void } q_j$; add these rules to \mathcal{V} ;

We note that the worst-case complexity of constructing \mathcal{V} is $O(N \times M)$ where N is the number of primitive transformations in the pathway and M is the maximum number of schema constructs appearing in comprehension expressions.

Once constructed, \mathcal{V} can be used to reformulate a query q posed on S_x with respect to S_y . We term a schema construct c which appears in S_y **final** otherwise it is **non-final**. The query reformulation algorithm is as follows, where the function $NF(q)$ returns the set of non-final schemes occurring in an IQL query q :

```

while  $NF(q) \neq \emptyset$ 
  for each  $c \in NF(q)$ 
     $e := \text{Range Void Any}$ 
    for each rule  $r \in \mathcal{V}$  such that  $\text{head}(r) = c$ 
       $e := \text{merge } e \text{ body}(r)$ 
     $q := [c/e]q$ 

```

In other words, non-final constructs in q are successively replaced by their definition in \mathcal{V} until there are no non-final constructs left. It is easy to see that this process terminates: Let \mathcal{G} be the graph obtained from \mathcal{V} by creating a node in \mathcal{G} for each schema construct in the head of a rule in \mathcal{V} and an arc $c \rightarrow c'$ in \mathcal{G} if c' appears in a rule defining c . The acyclicity of \mathcal{G} follows from the syntactic properties of BAV transformation sequences: an **add** or **extend** transformation can only add a construct that does not exist in the input schema, and the query within the transformation can only refer to constructs existing in the input schema; a **delete** or **contract** transformation can only delete a scheme that exists in the input schema and the query within the transformation can only refer to schemes existing in the output schema. By the acyclicity of \mathcal{G} the query reformulation algorithm must terminate. The complexity of the query reformulation algorithm is again $O(N \times M)$. The resulting query would then be optimised, applying equivalences such as those listed in Appendix A and illustrated in the earlier examples, and evaluated.

4 Data Source Schema Query Processing

BAV pathways can in principle be used to map directly between peer schemas in a P2P data integration scenario, and the techniques we have described above can be used to reformulate queries with respect to a BAV pathway between two peer data source schemas. However, defining pairwise mappings between peer data source schemas does not scale as the number of schemas grows. Thus, in AutoMed we also support P2P BAV data integration via **public schemas**, as already described in the Introduction.

A desirable property in data integration is that the mapping between a pair of schemas S_x and S_y should form a **complete mapping**, in the sense that it identifies all possible mappings between schema objects in schemas S_x and S_y . In our P2P framework as described earlier, we can construct mappings between schemas S_x and S_y by finding some shared or public schema S_z for which we already know the pathways $S_x \rightarrow S_z$ and $S_z \rightarrow S_y$, and form a concatenation of these two pathways to form a pathway $S_x \rightarrow S_y$.

However, this pathway may not in general represent a complete mapping, since S_z might not contain a schema object to represent data associated with schema objects that appear in S_x and S_y and for which a mapping could be specified in a *direct* pathway from S_x to S_y . Suppose that SO_x is a schema object in S_x and SO_y is a schema object in S_y for which a mapping between SO_x to SO_y could be established, but that it is currently absent due to the absence of a corresponding schema object in S_z . Then it must be the case that the pathway $S_x \rightarrow S_z$ contains a transformation of the form:

Ⓐ $\text{contractObj}_x(\text{SO}_x, \text{Range Void Any})$

expressing the fact that SO_x cannot be derived or represented in S_z , and similarly, $S_z \rightarrow S_y$ must contain a transformation of the form

Ⓒ extendObj_y(SO_y, Range Void Any)

expressing the fact that SO_y cannot be derived or represented in S_z .

However, the same pair Ⓐ, Ⓑ of transformations would exist if SO_x could not be derived or represented in S_z and S_y , and SO_y could not be derived or represented in S_z and S_x . Hence, we can use the presence of pairs of transformations of the form of Ⓐ and Ⓑ to extract pairs of schema objects that *might* be mappable between S_x and S_y , and feed such pairs into a **schema matching** process [RB01] in order to derive any mappings that exist between objects as yet unmapped in S_x and S_y . AutoMed supports a suitable interactive schema matching tool [Riz04]. This tool automatically derives possible matchings between pairs of schema objects, as well as the AutoMed transformations representing their mapping. The user is then asked to confirm or manually modify the matchings and generated transformations.

Thus, the procedure for constructing a complete mapping $S_x \rightarrow S_y$ from two complete mappings $S_x \rightarrow S_z$ and $S_z \rightarrow S_y$ is:

(i) Form the set USO_x of schema objects that appear in **contract** transformations in $S_x \rightarrow S_z$, and the set USO_y of schema objects that appear in **extend** transformations in $S_z \rightarrow S_y$.

(ii) Perform a pairwise match of objects in USO_x against objects in USO_y . For each positive match found, remove the transformation steps that contract/extend the matched pair of objects, and replace with the transformations that represent the match found.

To illustrate, we return to our running example. Within the pathway $S_3 \rightarrow S_p$ there are two transformations:

Ⓐ contractColumn(⟨⟨degree,qual⟩⟩, Range Void Any)

Ⓑ contractColumn(⟨⟨reg,tutor⟩⟩, Range Void Any)

When deriving the pathway $S_2 \rightarrow S_3$ from the pathway $S_2 \rightarrow S_p$ (which will include transformation Ⓐ) and the reverse of pathway $S_3 \rightarrow S_p$ (transformations Ⓒ–Ⓓ) — the filled in circles indicate that the reverse of the listed transformation is being used), a schema match table as follows is first formed:

Data Source S_2		Data Source S_3	
Transformation	Schema Object	Transformation	Schema Object
Ⓐ	⟨⟨teach,tname⟩⟩	Ⓐ	⟨⟨degree,qual⟩⟩
		Ⓑ	⟨⟨reg,tutor⟩⟩

The schema matching process should then discover that ⟨⟨teach,tname⟩⟩ and ⟨⟨reg,tutor⟩⟩ match (specifically, that they are equivalent, with the exception of the key used). Hence transformations Ⓐ and Ⓑ can be removed and the following transformations added to the end of $S_2 \rightarrow S_3$:

Ⓒ addColumn(⟨⟨reg,tutor⟩⟩, [{{s, c}, tu} | {{s, c, t}, tu} <- ⟨⟨teach,tname⟩⟩])

Ⓓ deleteColumn(⟨⟨teach,tname⟩⟩, [{{s, c, Const1 s c}, tu} | {{s, c}, tu} <- ⟨⟨reg⟩⟩; (Const1 s c) >= '1994-1'])

5 Concluding remarks

The BAV approach has the advantage in a P2P data integration setting of allowing bidirectional logical mappings to be specified between peers. We have shown how these

mappings can be used to support two types of query processing in a P2P data integration system, where either queries are posed on the schema of a data source at a peer or on a virtual public schema. We have also shown how GAV and LAV query reformulation can be combined over BAV pathways — specifically, for a comprehensions-based query language — thus obtaining the maximal information from BAV pathways that would be derivable by means of GAV and LAV query processing techniques.

We have focused here on query processing along a single BAV pathway, which cannot generate cyclic relationships between schema objects and hence for which query answering is decidable c.f. [HIST03]. The extension of P2P query processing along a network of arbitrary BAV pathways is an area of ongoing work, and in particular we wish to investigate the applicability of the epistemic semantics approach of [CDD⁺03,CDLR04] to our BAV setting.

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Appendix A - Some IQL Equivalences

(a) The following equivalence states that flatmap propagates through expressions of the form $\text{Range } e1 \ e2$:

$$\text{flatmap } f \ (\text{Range } e1 \ e2) = \text{Range} \ (\text{flatmap } f \ e1) \ (\text{flatmap } f \ e2)$$

(b) For any f :

$$\begin{aligned} \text{flatmap } f \ [] &= [], \text{ flatmap } f \ \text{Void} = \text{Void}, \text{ flatmap } f \ \text{Any} = \text{Any} \\ \text{flatmap} \ (\lambda x. []) \ e &= [], \text{ flatmap} \ (\lambda x. \text{Void}) \ e = \text{Void} \\ \text{flatmap} \ (\lambda x. \text{Any}) \ e &= \text{Any} \end{aligned}$$

(c) For any series of qualifiers Q and Q' , pattern p and expression e :

$$\begin{aligned} [e \mid Q; p \leftarrow []; Q'] &= [], [e \mid Q; p \leftarrow \text{Void}; Q'] = \text{Void}, [e \mid Q; p \leftarrow \text{Any}; Q'] = \text{Any} \\ [e \mid Q; \text{False}; Q'] &= [] \end{aligned}$$

(d) There are several other equivalences that can be used to simplify comprehension queries (see [PS96] for a comprehensive discussion). For example,

$$\begin{aligned} [e \mid Q; p1 \leftarrow e1; e2; Q'] &= [e \mid Q; e2; p \leftarrow e1; Q'] \\ [e \mid Q; e1 \ \text{and} \ e2; Q'] &= [e \mid Q; e1; e2; Q'] \end{aligned}$$

The first of these holds provided $\text{FV}(p1) \cap \text{FV}(e2) = \{\}$, where $\text{FV}(e)$ denotes the set of free variables in an expression e . From the second equivalence, it follows that if $e1$ and $e2 \equiv \text{False}$ then for any $e, Q, Q', [e \mid Q; e1; e2; Q'] = []$

Similarly, if $e1$ implies $e2$, then for any $e, Q, Q', [e \mid Q; e1; e2; Q'] = [e \mid Q; e1; Q']$

The following equivalence simplifies a nested comprehension:

$$[e \mid Q; p \leftarrow [p \mid p \leftarrow e']; Q'] = [e \mid Q; p \leftarrow e'; Q']$$

The following equivalence allows a filter to be moved inside a nested comprehension provided that $\text{FV}(e') \subseteq \text{FV}(p)$ and the pattern p' is more specific than the pattern p i.e. p' can be obtained from p by means of some substitution. The filter e'' is obtained from e' by substituting each variable from p in e' by its counterpart in p' (i.e. e'' is equivalent to $(\lambda p. e') \ p'$):

$$[e \mid Q; p \leftarrow [p' \mid Q]; e'; Q'] = [e \mid Q; p \leftarrow [p' \mid Q; e'']; Q']$$

(e) The following equivalences govern the simplification of expressions involving Range :

$$\begin{aligned} \text{Range} \ (\text{Range } e1 \ e2) \ \text{Any} &= \text{Range } e1 \ e2 \\ \text{Range } \text{Void} \ (\text{Range } e1 \ e2) &= \text{Range } e1 \ e2 \end{aligned}$$