Performance Trees: A New Approach to Quantitative Performance Specification

Tamas Suto, Jeremy T. Bradley, William J. Knottenbelt

Department of Computing
Imperial College London
United Kingdom

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Existing techniques for specifying performance queries are based on stochastic logics.

Steep learning curve – not straightforward to use.

Designed for model checking – constrained expressiveness.

Opportunities for improvement:

- Need to ease usability
- Need to expand scope of expressiveness
A Way to Go

- Maintain and expand expressiveness of stochastic logics
- Do not restrict to model checking-specific queries – incorporate quantitative analysis capabilities
- Use a graphical specification mechanism instead of logic
- Design a new formalism, incorporating all of the above, independent of stochastic logics
Query Specification with CSL

- Traditional performance query specification is based on CSL
- Operates on the state level (on CTMCs)

**Definition**

\[
\sigma \overset{\text{def}}{=} tt \mid a \mid \neg \sigma \mid \sigma \land \sigma \mid S_{\preceq p}(\sigma) \mid P_{\preceq p}(\varphi)
\]

\[
\varphi \overset{\text{def}}{=} X^l \sigma \mid \sigma U^l \sigma
\]

- Can express steady-state, path-based and nested constraints
“Is the probability of the system being down within 10 time units after having continuously operated with at least two processors at most 1%, when starting from state $s_3$?”

$$\text{Sat}(s_3) \models P_{\leq 0.01}((up_3 \lor up_2) \cup [0,10] \downarrow)$$
Example

“Is the probability of the system being down within 10 time units after having continuously operated with at least two processors at most 1%, when starting from state $s_3$?”

$$\text{Sat}(s_3) \models P_{\leq 0.01}((\text{up}_3 \lor \text{up}_2) \cup [0,10] \text{down})$$
Query Specification with CSL

Example

“Is the probability of the system being \textit{down} within 10 time units after having continuously operated with at least two processors at most 1\%, when starting from state $s_3$?”

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Query Specification with CSL

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Query Specification with aCSL

- An augmented version of CSL that allows the incorporation of actions into performance queries
- Operates on the state level (on CTMCs)

### Definition

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\[
\varphi \overset{\text{def}}{=} \sigma_A \mathcal{U}^t \sigma \mid \sigma_A \mathcal{U}^t_B \sigma
\]

- Can express steady-state, path-based and nested constraints with action restrictions
Example

“Having started in state $s_1$, is there a chance of more than 90% that the system will recover from a failure within the next 5 time units?”

$$\text{Sat}(s_1) \models \text{fail } P_{>0.9}(true_{\text{repair}} U_{\text{repair}}^{<5} true)$$
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Query Specification with aCSL

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Query Specification with eCSL

- An enhanced version of CSL that separates state and performance specifications
- Operates on the model level (on SM-SPNs)

**Definition**

\[
\sigma \overset{\text{def}}{=} tt \mid \neg \sigma \mid \sigma \land \sigma \mid p[N]
\]

\[
\varphi \overset{\text{def}}{=} tt \mid \neg \varphi \mid \varphi \land \varphi \mid S_\rho(\sigma) \mid T^\tau_\rho(\sigma, \sigma) \mid P^\tau_\rho(\sigma, \sigma)
\]

- Can express steady-state, transient and passage time requirements
Example

"Is the probability of the system being in the set of states defined by $p_5[2] \land p_6[1]$ at time instant 16 within the interval $[0.9, 0.95]$, having started from the set of states $p_1[10]$?"

$$\text{Sat}(p_1[10]) \models T_{[0.9,0.95]}^{[16,16]}(p_5[2] \land p_6[1])$$
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Query Specification with Performance Trees

- Graphical query representation formalism
- Offers full expressiveness of CSL, aCSL and eCSL
- Incorporates measure extraction capabilities
- Can deal with high-level concepts that were not expressible previously (such as moments, convolutions, distributions, densities, etc.)
- Take the form of a tree structure with operator and value nodes
Example

“Does a passage between the set of states $S$ and the set of states $T$ occur within the time interval $[0, 10]$ with probability in the range $[0.9, 0.98]$?”
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Example

“Does a passage between the set of states S and the set of states T occur within the time interval [0, 10] with probability in the range [0.9, 0.98]?”

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Stochastic Logics

Performance Trees
Example

“Does a passage between the set of states $S$ and the set of states $T$ occur within the time interval $[0, 10]$ with probability in the range $[0.9, 0.98]$?”
Example

“Is the probability that the system will be in the set of states $T$ at time instant 40, given that the system has originally started from the set of states $S$, greater than 0.87?”
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“Out of the set of states $S$, which states have a steady-state probability greater than 0.12?”
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```
SS:S

States

S

Num

0.12 prob.

Num

1 prob.
```

Suto, Bradley, Knottenbelt
Example

“What is the productivity of the system, defined as the sum of the mean firing rate of action ‘processed at A’ multiplied by 100, and the mean firing rate of action ‘processed at B’ multiplied by 200?”
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Example

“What is the variance of the passage time defined over the set of start states $S$ and the set of target states $T$, with the constraint that action ‘processed’ takes place at least once and that the action ‘halt’ does not occur during the passage?”
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Example

“What is the average time required to complete the passage defined by the convolution of the passage from the set of start states $S_1$ to the set of target states $T_1$ with the passage from the set of start states $S_2$ to the set of target states $T_2$, having the additional constraint that the set of states $E$ is excluded from both passages?”
Example

“What is the **average time** required to complete the passage defined by the convolution of the passage from the set of start states $S_1$ to the set of target states $T_1$ with the passage from the set of start states $S_2$ to the set of target states $T_2$, having the additional constraint that the set of states $E$ is excluded from both passages?”
Example

“What is the average time required to complete the **passage defined by the convolution** of the passage from the set of start states $S_1$ to the set of target states $T_1$ with the passage from the set of start states $S_2$ to the set of target states $T_2$, having the additional constraint that the set of states $E$ is excluded from both passages?”
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Query Specification with Performance Trees

1. Operation Nodes

- ? : result of query
- ; : sequential evaluation
- \( \land \) : conjunction or disjunction
- \( \neg \) : negation
- \( \bowtie \) : binary comparison (\(<, \leq, =, \geq, >\))
- \( \oplus \) : arithmetic operation (\(+, -, \times, \div\))
- PTD : passage time density
- Dist : passage time distribution
- Conv : convolution
- Moment : higher moment
- SS:P : steady-state probability of a set of states
- SS:S : set of states having a certain steady-state prob.
- FR : firing rate of a transition / action
Query Specification with Performance Trees

1. Operation Nodes (ctd.)

   - **InInterval**: is numerical value in a given range?
   - **ProbInInterval**: prob. with which passage occurs in a certain amount of time
   - **ProbInStates**: transient probability measure
   - **StatesAtTime**: set of states that the system is occupying at a given time with a certain probability

2. Value Nodes

   - **[. . .]**: numerical interval
   - **States**: set of states
   - **Actions**: set of actions
   - **Num**: numerical value
   - **Bool**: boolean value
Stochastic Logics vs. Performance Trees

Stochastic Logics

- One-inch logics – very concise, yet very restricted
- Hard to work with them – require expert understanding
- Lack a desired level of expressiveness
- Are not equipped for measure extraction
Stochastic Logics vs. Performance Trees

Performance Trees

- For most people, more natural to specify queries as PTs than as SLs
- Based on basic background knowledge of most engineers
- Subsume CSL, aCSL and eCSL
- Can represent a large variety of performance queries
- Have the notion of a state, which can be adapted to various modelling formalisms
- Extensive syntax, type system and semantics
Work in Progress

- Tool development
  - Performance Tree front-end
  - Integrate various existing toolkits into a meta-tool, since necessary numerical techniques already exist
  - Grid back-end for evaluation engine to cope with very large models
Thank you for your attention.

Any questions?