Stochastic Ambient Logic

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Abstract

We introduce the Stochastic Ambient Logic which extends the Ambient Logic with the probability operator and the steady state operator. The logic is useful for specifying properties of the Stochastic Mobile Ambients [14].

Keywords: Process algebra, Markov chains, Stochastic modelling

1 Introduction

In this paper we will present an extension of Mobile Ambients MA with continuous time delays and probabilities, where the underlying model is Markovian. We call this calculus Stochastic Mobile Ambients (SMA).

This paper is a continuation of previous work [14], where we presented the semantics in terms of labelled transition system and we defined a strong Markovian bisimulation. In the current paper, however, we define the semantics based on the reduction relation and we extend the Ambient Logic [4] with the probability operator and the steady state operator. We call this logic Stochastic Ambient Logic (SAL).

SMA is a standard extension of MA [7,11,10]. We augment the prefix of the capabilities with a *rate name* r as in [10] i.e. $\mathsf{in}\,(n,r), \mathsf{out}\,(n,r), \mathsf{open}\,(n,r)$ so that different transitions of the same process remain distinct. To each rate name r, a *rate* $\lambda \in \mathbb{R}$ is associated via a function. In this way, we make sure that each transition in the calculus is delayed by a random amount of time sampled from the negative exponential distribution. To each ambient a function f is associated that influences the rate of the computation, i.e. $n[P]^f$. This means that we can regard each ambient as running at a particular speed. The Stochastic Ambient Logic allows to express property on the SMA.

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We consider the logic without the constructors for dealing with hidden names, however our work could be easily extended to include the full Ambient Logic [5]. Our work on the logic can be used to express properties on other Ambient Calculus dialects [8] and to express properties on biological modelling [12].

2 Stochastic Mobile Ambients

We assume the existence of a *set of names* $\mathcal{N}$ ranged over by $a, n, m, \ldots$, and a set of *rate names* $\mathcal{R}$ ranged over by $r, r', r'', \ldots$. Moreover, we assume that the above sets are mutually disjoint.

Definition 2.1 The set of *process terms* of SMA is given by the following syntax:

$$
\begin{array}{rcll}
P, Q & ::= & \mathbf{0} & \textit{nil} \\
 & \mid & \sum_{i \in I} M_i P_i & \textit{local sum} \\
 & \mid & n[P]^f & \textit{stochastic ambient} \\
 & \mid & P \mid Q & \textit{composition} \\
 & \mid & (\nu n)P & \textit{restriction} \\
 & \mid & (\mathit{fix}\ A = P) & \textit{recursion} \\
 & \mid & A & \textit{identifier}
\end{array}
$$

where I is a set of indexes and M stands for the *capabilities* defined by the following grammar:

$$
\begin{array}{rcll}
M, N & ::= & \mathsf{in}\,(n, r) & \textit{enter capability} \\
 & \mid & \mathsf{out}\,(n, r) & \textit{exit capability} \\
 & \mid & \mathsf{open}\,(n, r) & \textit{open capability}
\end{array}
$$

Nil represents the inactive process; *Local sum* $\sum_{i \in I} M_i.P_i$ represents the standard choice; we assume the existence of a binding function $\ell : \mathcal{R} \to \mathbb{R}$ such that in the prefixes $M.P$ it is possible to extract via ℓ the rate of the random delays of the transitions. Each name rate is unique in each prefix of a given process. Given a set of indexes I and a permuatation p on it, we write $\sum_{p(i) \in I} M_{p(i)}.P_p(i)$ to represent a reordering of the terms of the summation. *Stochastic ambient* $n[P]^f$ is composed of the name of the ambient n, the active process P, and the function f $f : \mathbb{R} \to \mathbb{R}$. *Parallel composition* $P \mid Q$ means that P and Q are running in parallel. *Restriction* $(\nu a)P$ of the name n makes that name private and unique to P. *Recursion* $(\mathit{fix}\ A = P)$ models infinite behaviour.

We define here semantics in terms of a reduction relation. The main reason for working on reduction semantics is that labelled semantics for MA is very complicated [14], and reduction semantics provides a means to deal with computation in a simpler way. We present the reduction semantics in line with previous work on MA [4], since it also adapts well to the work on the logic.

$$P \mid \mathbf{0} \equiv P \qquad \text{(Struct Par Zero)}$$

$$P \mid Q \equiv Q \mid P \qquad \text{(Struct Par Comm)}$$

$$(P \mid Q) \mid R \equiv P \mid (Q \mid R) \qquad \text{(Struct Par Assc)}$$

$$(\nu n)\mathbf{0} \equiv \mathbf{0} \qquad \text{(Struct Restr Zero)}$$

$$(\nu m)(\nu n)P \equiv (\nu n)(\nu m)P \qquad \text{(Struct Restr Comm)}$$

$$(\nu n)(P \mid Q) \equiv P \mid (\nu n)Q \quad \text{if } n \notin \mathsf{fn}(P) \qquad \text{(Struct Restr Par)}$$

$$(\nu m)n[P]^f \equiv n[(\nu m)P]^f \quad \text{if } n \neq m \qquad \text{(Struct Restr Amb)}$$

$$\textstyle\sum_{i\in I} M_i.P_i \equiv \sum_{p(i)\in I} M_{p(i)}.P_{p(i)} \qquad \text{(Struct Choice)}$$

$$(\mathit{fix}\ A = P) \equiv P\{A/(\mathit{fix}\ A = P)\} \qquad \text{(Struct Fix Rec)}$$

Fig. 1. Structural congruence

$$\text{(RED IN)} \qquad m[\mathsf{in}\,(n,r).P + S \mid Q]^f \mid n[R]^{f'} \xrightarrow{r} n[m[P \mid Q]^f \mid R]^{f'}$$

$$\text{(RED OUT)} \qquad n[m[\mathsf{out}\,(n,r).P + S \mid Q]^f \mid R]^{f'} \xrightarrow{r} m[P \mid Q]^f \mid n[R]^{f'}$$

$$\text{(RED OPEN)} \qquad \mathsf{open}\,(n,r).P + S \mid n[P]^f \xrightarrow{r} P \mid Q$$

$$\frac{P \xrightarrow{r} P'}{P \mid R \xrightarrow{r} P' \mid R}\ \text{(RED PAR)} \qquad \frac{P \xrightarrow{r} P'}{(\nu n)P \xrightarrow{r} (\nu n)P'}\ \text{(RED RESTR)}$$

$$\frac{P \xrightarrow{r} P'}{n[P]^f \xrightarrow{f(r)} n[P']^f}\ \text{(RED AMB)} \qquad \frac{P \equiv P' \xrightarrow{r} Q' \equiv Q}{P \xrightarrow{r} Q}\ \text{(RED CONG)}$$

Fig. 2. Reduction Relation

Definition 2.2 The *Structural Congruence Relation* over the SMA is the smallest relation satisfying the equations in Figure 1.

Definition 2.3 The *reduction relation* $\longrightarrow\ \subseteq \mathsf{SMA} \times \mathcal{R} \times \mathsf{SMA}$ is the smallest relation satisfying the set of rules in Figure 2.

To the reduction relation we can associate a transition relation $\to' SMA \times \mathbb{R} \times SMA$ such that if $P \xrightarrow{r} P'$ then $P \xrightarrow{\lambda_{(P,P')}}{}' P'$ where $\lambda_{(P,P')} \stackrel{\text{df}}{=} \sum_{P \xrightarrow{r} P'} \ell(r)$ or if $P \xrightarrow{f(r)} P'$ then $P \xrightarrow{\lambda_{(P,P')}}{}' P'$ where $\lambda_{(P,P')} \stackrel{\text{df}}{=} \sum_{P \xrightarrow{f(r)} P'} f(\ell(r))$. We will show how to associate a Markov Chain to each process in SMA.

Definition 2.4 An *unlabelled Markov chain* (MC) $\mathcal{M}$ is a pair $\mathcal{M} = \langle S, \to \rangle$ where S is a set of states, and $\to \subset S \times \mathbb{R} \times S$ is the transition relation.

We shall assume that any MC is finite i.e. has finite number of states and it is finitely branching. We let $s, s' \ldots$ range over S. The transition $s \xrightarrow{\lambda} s'$ denotes the system moving from state s to state s' after a random delay determined from the negative exponential distribution with rate λ. We write $\mathbf{R}(s,s') \stackrel{\mathrm{df}}{=} \sum_{s \xrightarrow{\lambda} s'} \lambda$ to denote the cumulative rate of moving from state s to state s'; we write $\mathbf{E}(s) \stackrel{\mathrm{df}}{=} \sum_{s' \in S} \mathbf{R}(s,s')$ the total rate of possible transition from state s and $\mathbf{p}(s,s') \stackrel{\mathrm{df}}{=} \mathbf{R}(s,s')/\mathbf{E}(s)$ as the proabability of changing state. An absorbing state s i such that $\mathbf{E}(s) = 0$ and $\mathbf{p}(s,s') = 0$. In this paper we considering only finite state SMA, and we can regard this calculus as an unlabelled Markov chain $\mathcal{M} \stackrel{\mathrm{df}}{=} \langle \mathsf{SMA}, \longrightarrow \rangle$.

3 Stochastic Ambient Logic

In this section we will extend the Ambient Logic [4,5] to the probabilistic setting. We take the standard approach in probabilistic temporal logics (see for example PCTL [6,3] and CSL [1,2]) and replace the *some* time operator $\Diamond\phi$ with a probabilistically quantified version of the form $\mathsf{IP}_{\bowtie p}(\Diamond\phi)$ where $\bowtie \in \{<, \leq, \geq, >\}$ and $p \in [0,1]$ and we introduce the standard steady state operator $\mathsf{S}_{\bowtie p}(\phi)$. In the following we have not considered the logical operators that deal with hidden names as [5], for reasons of simplicity in the presentation. The syntax of the Probabilistic Ambient Logic (SAL) is defined below.

Definition 3.1 The set of logical formulae of the Ambient Logic, written PAL, is the smallest set defined below:

$$\phi, \psi ::= \mathbf{T} \;\mid\; \neg\phi \;\mid\; \phi \vee \psi \;\mid\; \mathbf{0} \;\mid\; n[\phi] \;\mid\; \phi \,|\, \psi \;\mid\; \maltese\, \phi \;\mid\; \phi@n \;\mid\; \phi \triangleright \psi$$

$$\mathsf{IP}_{\bowtie}(\Diamond\phi) \quad \text{probabilistic operator}$$

$$\mathsf{S}_{\bowtie}(\phi) \quad \text{steady state operator}$$

The first three connectives are standard in propositional logic. The remaining connectives are tailored to express properties about ambient processes relative to both *time* and *space*. $n[\phi]$ expresses that *here and now* there is an ambient called n inside which ϕ holds. $\maltese\,\phi$ expresses that *some where*, i.e. after traversing down through a number of ambients, ϕ holds. $\phi@n$ expresses that *in context* n, i.e. after being placed inside the ambient n, ϕ holds. $\phi \triangleright \psi$ expresses that ψ holds in *in any context satisfying* ϕ. Intuitively, process satisfies such a formula $\mathsf{IP}_{\bowtie p}(\Diamond\phi)$ if the probability of reaching a process satisfying the formula ϕ satisfies the condition $\bowtie p$. Similarly a process satisfies the formula $\mathsf{S}_{\bowtie p}(\phi)$ if the steady-state probability of a set of states satisfying ϕ meets the bound of p

Definition 3.2 Let $P \in \mathsf{SMA}$. The process P' is *a step away* from P, written $P \downarrow P'$ iff $\exists n, P''$ such that $P \equiv n[P']^f \mid P''$.

We write $\downarrow^*$ for the reflexive and transitive closure of the $\downarrow$ relation.

$P \models \mathbf{T}$	iff always
$P \models \neg\phi$	iff not $P \models \phi$
$P \models \phi \vee \psi$	iff $P \models \phi$ or $P \models \psi$
$P \models \mathbf{0}$	iff $P \equiv \mathbf{0}$
$P \models n[\phi]$	iff there exists P' such that $P \equiv n[P']^f$ and $P' \models \phi$
$P \models \phi \mid \psi$	iff there exist P', P'' such that $P \equiv P' \mid P''$ and $P' \models \phi$ and $P' \models \psi$
$P \models \between \phi$	iff there exists P' such that $P \downarrow^* P'$ and $P' \models \phi$
$P \models \phi@n$	iff $n[P]^f \models \phi$
$P \models \phi \triangleright \psi$	iff for all P' if $P' \models \phi$ then $P \mid P' \models \psi$
$P \models \mathsf{IP}_{\bowtie p}(\Diamond\phi)$	iff $\mathit{Prob}(\{\pi \in \mathit{Path}(P) : \pi \models \Diamond\phi\}) \bowtie p$
$P \models \mathsf{S}_{\bowtie p}(\phi)$	iff $\lim_{t\to\infty} \mathit{Prob}(\{\pi \in \mathit{Path}(P) : \pi(t) \models \phi\}) \bowtie p$

$$\text{where} \quad \pi \models \Diamond\phi \quad \text{iff} \quad \exists i \quad \pi(i) \models \phi.$$

Fig. 3. Stochastic Ambient Logic

Definition 3.3 Let $\mathcal{M}$ be a Markov Chain. An *infinite path* π over SMA is a sequence $\pi \stackrel{\text{df}}{=} s_0 \xrightarrow{t_0} s_1 \xrightarrow{t_1} s_2 \xrightarrow{t_2} \cdots \xrightarrow{t_i} s_{i+1} \cdots$ such that $\forall i$ that $i \geq 0$ $s_i \in S$ $t_i \in \mathbb{R}^+$ and $\mathbf{R}(s_i, s_{i+i}) > 0$. We write $\pi(i)$ for the ith state in the path π and $\Delta(\pi, i) = t_i$ is the time spent in state s_i. For $t \in \mathbb{R}$ and i the smallest index such that $t \leq \sum_{0 \leq j \leq i} t_j$ we let $\pi(t) = \pi(i)$. Finite paths are defined similarly with an ending state being absorbing.

The set of *paths* (finite and infinite) starting in s is denoted by $\mathit{Path}(s)$. A Borel space can be defined over $\mathit{Path}(s)$ together with the associated probability measure *Prob*.

Definition 3.4 The satisfaction relation $\models$: SMA $\times$ SAL, written $P \models \phi$ is the smallest relation defined in Figure 3.

It has been shown [9,13] that the equivalence relation induced by the ambient logic roughly coincides with structural congruence - for the finite fragment of Mobile Ambients. We leave to future investigation to show whether this still hold in the SAL.

4 Conclusions

As future work is concerned we envisage to show that the current reduction semantics and the labelled semantics defined in [14] are the same. We also aim to apply both the Stochastic Ambient Calculus and the Stochastic Ambient logic to model and verify queues.

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