

ROBUST MULTIUSER DETECTION IN FLAT FADING NON-GAUSSIAN CHANNELS

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ABSTRACT

This paper deals with the problem of multiuser detection (MUD) in direct-sequence code-division multiple-access (DS/CDMA) fading channels with impulsive noise. This issue is of importance because in many practical situations the ambient noise is impulsive due to various natural and man-made impulsive sources. In this paper a novel H^∞ estimation based robust multiple model detector (RMMD) is proposed. We have considered here binary phase-shift keyed (BPSK) signals transmitted simultaneously by multiple users via a CDMA flat-fading channel and embedded in impulsive noise. Simulation results show that, in impulsive noise and low signal-to-noise ratio (SNR), the bit-error-rate (BER) performance gain afforded by the proposed multiuser detector can be substantial when compared to a H^2 based multiple model detector (IMM-SIC) and the iterative M -estimator-based decorrelating detector, recently proposed in the literature.

1. INTRODUCTION

In many physical channels, such as urban and indoor radio channels and underwater acoustic channels, the ambient noise is known to be non-Gaussian, due to the impulsive man-made electromagnetic interference and natural noise as well. Thus the significant interest in demodulation techniques for non-Gaussian multiple-access channels.

The optimum (in the maximum-likelihood (ML) sense) multiuser detector for data detection in multiple access non-Gaussian channels has been derived in [1] and its performance is shown to be substantial compared to the optimum ML detector based on the Gaussian assumption [2]. Since the ML strategy is computationally intensive a lower complexity detector has been proposed in [3], [4] for demodulating multiuser signals in the presence of both multiple-access interference (MAI) and non-Gaussian ambient noise. These recent methods are based on the M -estimation method for robust regression and is essentially an iterative robustified version of the linear decorrelating multiuser detector.

The novel idea proposed in this paper is a multiple model structure employing a worst-case deterministic cost for data decoding and a linear successive cancellation for mitigating MAI. The deterministic performance index at each time-instant is derived from a norm bounded transfer operator which minimizes the estimation error with respect to worst-case measurement noise, driving noise and initialization error, effectively reducing the effects of impulsive noise based on a min-max strategy.

2. SYSTEM MODEL

In this paper we consider a CDMA system in which the signal of each of K users arrives at the receiver through an independent, single-path, flat fading channel, so that the received signal can be written as

$$r(t) = \sum_{k=1}^K \sum_{i=1}^M A_k b_k[i] c_k(t) s_k(t - iT - \tau_k) + \varpi(t) \quad (1)$$

M is the number of symbols per user in the data frame of interest, T is the symbol interval, $c_k(t)$, τ_k , and $\{b_k[i] : i = 1, \dots, M\}$ denote, respectively, complex channel fading process, the propagation delay and the symbol stream for the k th user, and $\varpi(t)$ represents the ambient channel noise. It is assumed that $b_k[i] \in \{-1, 1\}$, and that the modulation waveforms $s_k(t)$ are zero for $t \notin [0, T]$. Each user is assigned an orthogonal spreading sequence of the form

$$s_k(t) = \sum_{n=1}^N s_n^k p_{T_c}(t - (n-1)T_c). \quad (2)$$

Here, $s_1^k, s_2^k, \dots, s_N^k$ is a signature sequence of $+1$ s and -1 s assigned to the k th user, and p_{T_c} is a unit-amplitude pulse of duration T_c (where $NT_c = T$). Assuming that the fading process for each user varies at a slow enough rate that the real and complex components of channel gain can be taken to be constant over the duration of a bit, and restricting, for simplicity, to the synchronous case (i.e., $\tau_1 = \tau_2 = \dots = \tau_K = 0$), it follows that,

$$r_n[i] = \sum_{k=1}^K A_k b_k[i] c_k[i] s_n^k + \varpi_n[i], \quad n = 1, \dots, N \quad (3)$$

$\{\varpi_n[i]\}$ is a sequence of independent and identically distributed (i.i.d.) complex random variables whose in-phase and quadrature components are independent non-Gaussian random variables with a common probability density function (pdf). The widely used two-term Gaussian mixture model is used, that is for the real component,

$$\Re\{\varpi_n[i]\} \sim (1 - \epsilon)\mathcal{N}(0, \sigma_n^2/2) + \epsilon\mathcal{N}(0, \sigma_I^2/2) \quad (4)$$

where $\sigma_I^2 = \kappa\sigma_n^2$ with $0 < \epsilon < 1$ and $\kappa > 1$. Here $\mathcal{N}(0, \sigma_n^2/2)$ represents the nominal ambient noise and the term $\mathcal{N}(0, \sigma_I^2/2)$ represents an impulsive component. The probability that impulses occur is ϵ . The overall complex noise variance is

$$\rho^2 \triangleq (1 - \epsilon)\sigma_n^2 + \epsilon\sigma_I^2 \quad (5)$$

3. PROPOSED H^∞ -BASED ROBUST MULTIPLE MODEL DETECTOR (RMMD)

With channel fading and symbols transmitted unknown we employ a multiple model structure to estimate these values simultaneously. In order to introduce the multiple model detector for the MUD problem let us observe that the synchronous signal model (3) can be written in matrix notation as

$$\mathbf{r}[i] = \mathbf{S}\mathbf{A}\mathbf{B}[i]\mathbf{c}[i] + \varpi[i] \quad (6)$$

where

$$\begin{aligned} \underline{\mathbf{s}}_k &= [s_1^k, \dots, s_N^k]^\top, \quad \mathbf{S} \triangleq [\underline{\mathbf{s}}_1 \cdots \underline{\mathbf{s}}_K] \\ \mathbf{A} &\triangleq \text{diag}(A_1 \cdots A_K), \quad \mathbf{B} \triangleq \text{diag}(b_1 \cdots b_K) \\ \underline{\varpi} &= [\varpi_1^* \cdots \varpi_N^*]^\top \end{aligned} \quad (7)$$

Matched filtering yields,

$$\mathbf{y}[i] = \mathbf{R}\mathbf{A}\mathbf{B}[i]\underline{\mathbf{c}}[i] + \underline{\varpi}[i] \text{ where } \mathbf{R} = \mathbf{S}^\top \mathbf{S} \quad (8)$$

The idea behind multiple model detector will be to operate several filters in parallel each filter matched to a possible combination (hypothesis) of the data set $\{b_k[i]\}_{k=1}^K$ forming the symbol matrix $\mathbf{B}[i]$. Each model matched filter calculates a performance index (PI) and the estimate of the channel $\underline{\mathbf{c}}[i]$ is given by the filter with the best PI. The decoded symbol matrix will be given by the data set corresponding to the hypothesis with the best PI. The drawback with this optimal approach is that there will be S^K hypotheses at any given time where S is the size of the symbol alphabet and the complexity grows exponentially with the number of users and time. Therefore the following successive cancellation scheme is introduced to the detector structure, which also mitigates MAI. Cholesky factorization of the correlation matrix yields, $\mathbf{R} = \mathbf{U}\mathbf{L}$ where \mathbf{U}, \mathbf{L} are upper and lower triangular matrices respectively. Sending the matched filtered signal through filter \mathbf{U}^{-1} yields,

$$\underline{\mathbf{z}}[i] = \mathbf{L}\mathbf{A}\mathbf{B}[i]\underline{\mathbf{c}}[i] + \underline{\nu}[i] \quad (9)$$

where $\underline{\mathbf{z}}[i] = [z_1^*[i], \dots, z_K^*[i]]^*$, and,

$$\begin{aligned} z_k[i] &= \underbrace{\mathbf{L}_{k,1:k-1} \mathbf{A}_{1:k-1} \mathbf{B}_{1:k-1}[i] \underline{\mathbf{c}}_{1:k-1}[i]}_{MAI} \\ &\quad + L_{k,k} A_k b_k[i] c_k[i] + \nu_k[i] \end{aligned} \quad (10)$$

where,

$$\begin{aligned} \mathbf{L}_{k,1:k-1} &= [L_{k,1} \cdots L_{k,k-1}], \quad \mathbf{B}_{1:k-1} = \text{diag}(b_1 \cdots b_{k-1}) \\ \mathbf{A}_{1:k-1} &= \text{diag}(A_1 \cdots A_{k-1}), \quad \underline{\mathbf{c}}_{1:k-1} = [c_1^* \cdots c_{k-1}^*]^\top \end{aligned}$$

Now we can implement a multiple model based detector for each user and using the previous users' estimates the MAI can be eliminated successively. The number of hypotheses for each user in a BPSK system ($S = 2$) will be 2 and the complexity of the system is now linear in K . Simplifying the notation, we may write (10) as,

$$z_k[i] = H_k[i] c_k[i] + \underline{\mathcal{H}}_k[i] \underline{\mathcal{C}}_k[i] + \nu_k[i] \quad (11)$$

where,

$$\begin{aligned} H_k[i] &= L_{k,k} A_k b_k[i]; \quad \underline{\mathcal{C}}_k = \underline{\mathbf{c}}_{1:k-1}; \\ \underline{\mathcal{H}}_k[i] &= \mathbf{L}_{k,1:k-1} \mathbf{A}_{1:k-1} \mathbf{B}_{1:k-1}[i] \end{aligned}$$

The basic idea of the H^∞ -based MUD (RMMD) is to detect the symbol $b_k[i]$ in (11) by first associating a set of models with all possible values for $b_k[i]$ and the model which has the best PI in terms of a min-max strategy, developed in the next section, gives the value of $b_k[i]$. For a BPSK system the number of models will be $\mathcal{M} = S (= 2)$. Therefore the multiple model detector is implemented using 2 H^∞ filters operating in parallel. The discrete-time Riccati solution for H^∞ filtering, presented in [5], [6], is used to estimate the channel $\hat{c}_k[i|i]$ and the Riccati variable $P_k[i|i]$. Each model-matched filter will also calculate a robust performance measure $\Delta J_{i|i-1}(\cdot)$ with respect to worst case uncertainty (a min-max problem) which is the PI for the decoder. The model with the minimum $\Delta J_{i|i-1}(\cdot)$ gives the symbol estimate $\hat{b}_k[i]$.

3.1. Robust Performance Measure

In practice the physical channel model is rarely known with certainty at the receiver, in order to jointly detect the symbols and the channel gains the following state-space model is used,

$$\begin{aligned} c_k[i+1] &= F c_k[i] + w_k[i]; \quad c_k[0] = x_0 \\ z_k[i] &= H_k[i] c_k[i] + \underline{\mathcal{H}}_k[i] \underline{\mathcal{C}}_k[i] + \nu_k[i] \\ s_i &= \theta_k[i] c_k[i] \end{aligned} \quad (12)$$

for some known F , (typically with $F = f$ for some scalar $0 \ll f < 1$). $w_k[i]$ is the complex driving disturbance whose components are zero-mean random processes and x_0 is the initial channel estimate. $\theta_k \in \mathcal{R}$. Thus we estimate an arbitrary linear scaling, say s_i of the channel c_k given the measurement signal $z_k[i]$, of a multiuser system with non-Gaussian noise. The estimation error will be, $e_{k,i} = \tilde{s}_{i|i} - \theta_k[i] c_k[i]$. The H^∞ estimator minimizes the maximum energy gain from the worst-case disturbances to the estimation errors, and, the robustness of the estimator is due to this worst-case performance and not assuming any statistical model or distribution to the disturbance signals. The performance measure for sequence length M is denoted here with the transfer operator \mathcal{J} . Dropping the time index for clarity,

$$\mathcal{J} \triangleq \frac{\sum_{i=1}^M e_{k,i}^* e_{k,i}}{x_0^* P_0^{-1} x_0 + \sum_{i=1}^M w_k^* Q^{-1} w_k + \sum_{i=1}^M \nu_k^* \nu_k} \quad (13)$$

P_0, Q are positive definite weights, a design parameter of the system. $\nu_k[i] = (z_k[i] - \underline{\mathcal{H}}_k[i] \underline{\mathcal{C}}_k[i]) - H_k[i] c_k[i]$, i.e. the MAI term is a deterministic quantity which is deducted from the measured signal. The objective of H^∞ estimation is that the optimal estimate of $\{s_i\}_{i=1}^M$, out of all possible estimates $\{\tilde{s}_{i|i}\}_{i=1}^M$, should minimize the H^∞ norm of the transfer operator \mathcal{J} which is defined as,

$$\|\mathcal{J}\|_\infty \triangleq \sup_{x_0, w \in H^2, \nu \in H^2} \mathcal{J}, \quad (14)$$

where H^2 norm of a causal sequence $\{x_i\}$ is $\sum_{i=1}^M x_i^* x_i$. In this paper we address the bounded min-max problem of

$$\inf_{s_i} \|\mathcal{J}\|_\infty = \inf_{s_i} \sup_{x_0, w \in h^2, \nu \in h^2} \mathcal{J} < \gamma_f^2 \quad (15)$$

where $\gamma_f^2 > 0$ is a level of disturbance attenuation. The objective being to estimate $\{\tilde{s}_{i|i}\}_{i=1}^M$ such that the minimum of the H^∞ norm of \mathcal{J} is less than or equal to a prescribed positive value γ_f^2 . Now we develop the recursive calculation of the PI, $\Delta J_{i|i-1}(\cdot)$ for the multiple model detector.

3.2. Recursive Computation of Performance Index ($\Delta J_{i|i-1}$)

The condition (15) can be related to a positivity of an indefinite-quadratic form J_M for the sequence length M derived by substituting (13) in R.H.S. of (15),

$$J_M(x_0, w_k^M, z_k^M) = x_0^* P_0^{-1} x_0 + \sum_{i=1}^M w_k^*[i] Q^{-1} w_k[i] + \sum_{i=1}^M \begin{bmatrix} z_k[i] - \underline{H}_k[i] \underline{C}_k[i] - H_k[i] c_k[i] \\ \tilde{s}_{i,i} - \theta_k[i] c_k[i] \end{bmatrix}^* \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix} \begin{bmatrix} z_k[i] - \underline{H}_k[i] \underline{C}_k[i] - H_k[i] c_k[i] \\ \tilde{s}_{i,i} - \theta_k[i] c_k[i] \end{bmatrix} > 0 \quad (16)$$

and substituting for ν_k from (12) and for $e_{k,i}$, $w_k^M = \{w_k[i]\}_{i=1}^M$ and $z_k^M = \{z_k[i]\}_{i=1}^M$. The quadratic form is a function of only $\{x_0, w_k^M, z_k^M\}$ and the only free variables in J_M are $\{x_0, w_k^M, z_k^M\}$. Substituting $\underline{\zeta}_{k,i} = [[z_k[i] - \underline{H}_k[i] \underline{C}_k[i]]^*, s_{i,i}^*]^*$, $\underline{\Theta}_k = [H_k^*, \theta_k^*]^T$ and $\underline{\eta}_k = \underline{\zeta}_{k,i} - \underline{\Theta}_k c_k[i]$ in (16) we can write,

$$J_M(x_0, w_k^M, z_k^M) = x_0^* P_0^{-1} x_0 + \sum_{i=1}^M w_k^*[i] Q^{-1} w_k[i] + \sum_{i=1}^M (\underline{\zeta}_{k,i} - \underline{\Theta}_k[i] c_k[i])^* \begin{bmatrix} 1 & 0 \\ 0 & -\gamma_f^{-2} \end{bmatrix} (\underline{\zeta}_{k,i} - \underline{\Theta}_k[i] c_k[i]) > 0 \quad (17)$$

$$\text{and } \underline{\zeta}_{k,i} = \underline{\Theta}_k[i] c_k[i] + \underline{\eta}_k[i] \quad (18)$$

For the sequence length $i = 1, \dots, M$, we can write,

$$\underline{\mathbf{Y}}_k = \underline{\Omega} x_0 + \underline{\Gamma} \underline{\mathbf{w}}_k + \underline{\eta}_k \text{ and then the matrix form } \begin{bmatrix} x_0 \\ \underline{\mathbf{w}}_k \\ \underline{\mathbf{Y}}_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathbf{I}_M & 0 \\ \underline{\Omega} & \underline{\Gamma} & \mathbf{I}_{2M} \end{bmatrix} \begin{bmatrix} x_0 \\ \underline{\mathbf{w}}_k \\ \underline{\eta}_k \end{bmatrix} \quad (19)$$

where $\underline{\mathbf{Y}}_k = [\underline{\zeta}_{k,1}^*, \dots, \underline{\zeta}_{k,M}^*]^*$, $\underline{\mathbf{w}}_k = [w_k^*[1] \dots w_k^*[M]]^*$, $\underline{\eta}_k = [\underline{\eta}_k^*[1] \dots \underline{\eta}_k^*[M]]^*$, and $\underline{\Omega} \in \mathcal{R}^{2M}$, $\underline{\Gamma} \in \mathcal{R}^{2M \times M}$ are the observability map and the impulse response matrix, respectively. $\mathbf{I}_M \in \mathcal{R}^{M \times M}$ is an identity matrix. Making a change of variables using (19), we can write (17) as,

$$J_M(x_0, w_k^M, z_k^M) = \begin{bmatrix} x_0 \\ \underline{\mathbf{w}}_k \\ \underline{\mathbf{Y}}_k \end{bmatrix}^* \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathbf{I}_M & 0 \\ \underline{\Omega} & \underline{\Gamma} & \mathbf{I}_{2M} \end{bmatrix} \begin{bmatrix} x_0 \\ \underline{\mathbf{w}}_k \\ \underline{\eta}_k \end{bmatrix} \right\}^{-1} \begin{bmatrix} P_0 & 0 & 0 \\ 0 & \mathbf{I}_M \otimes Q & 0 \\ 0 & 0 & \mathbf{I}_M \otimes \mathbf{R} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathbf{I}_M & 0 \\ \underline{\Omega} & \underline{\Gamma} & \mathbf{I}_{2M} \end{bmatrix}^* \begin{bmatrix} x_0 \\ \underline{\mathbf{w}}_k \\ \underline{\mathbf{Y}}_k \end{bmatrix} \quad (20)$$

where, \otimes is the Kronecker product, $\mathbf{R} = \text{diag}(1, -\gamma_f^2)$. Stationary point $\underline{\mathcal{T}}_0$ of quadratic form $J_M(\underline{\mathcal{T}}, z_k^M)$, where $\underline{\mathcal{T}} = [x_0^*, \underline{\mathbf{w}}_k^*]^*$, is easily obtained by partial differentiation w.r.t. $\underline{\mathcal{T}}$, which gives $J_M^{min}(\underline{\mathcal{T}}_0, z_k^M) = \underline{\mathbf{Y}}_k^* \mathbf{R}_{\underline{\mathcal{T}},k}^{-1} \underline{\mathbf{Y}}_k$. $\mathbf{R}_{\underline{\mathcal{T}},k}$ is a coefficient matrix. But in order to satisfy condition (15) this minimum has to be positive for the estimates $\{\tilde{s}_{i|i}\}_{i=1}^M$, i.e., $J_M^{min}(\underline{\mathcal{T}}_0, z_k^M) > 0$.

A partially equivalent Krein Space model (corresponding with (12) and using (18)) can be defined (partial as Krein space variables are stochastic). Krein space variables are differentiated here with a bar.

$$\begin{aligned} \bar{c}_k[i+1] &= F \bar{c}_k[i] + \bar{w}_k[i]; \\ \bar{\zeta}_{k,i} &= \underline{\Theta}_k[i] \bar{c}_k[i] + \bar{\eta}_k[i] \end{aligned} \quad (21)$$

[7] has detailed Krein space state-space representation and development of inner product matrices which are *equivalent* to the coefficient matrices in the deterministic quadratic form (derived from model (12),(18)) described in (20). A recursive formula, based on innovations in an *equivalent* Krein space, which can recursively compute $J_M^{min}(\underline{\mathcal{T}}_0, z_k^M)$, are presented in detail in [7].

We summarize here and introduce some changes to fit with MAI cancellation and symbol-by-symbol detection of the robust detector (RMMD). The innovation of the deterministic quantities is defined as $\underline{e}_i = \underline{\zeta}_{k,i} - \hat{\zeta}_{k,i}$ where $\hat{\zeta}_{k,i} \triangleq \hat{\zeta}_{k,i|i-1}$. The latter can be expressed in terms of innovations as $\hat{\zeta}_{k,i|i-1} = \sum_{j=1}^i \langle \underline{\zeta}_{k,i}, \underline{e}_j \rangle \langle \underline{e}_j, \underline{e}_j \rangle^{-1} \underline{e}_j$ where the coefficient matrix of the deterministic form is the same as the inner-product matrices of the innovations in the stochastic Krein space, $\langle \hat{\zeta}_{k,i}, \hat{\zeta}_{k,i} \rangle = \langle \underline{e}_j, \underline{e}_j \rangle^{-1}$. Therefore we can express the vector composed of $\{\underline{\zeta}_{k,i}\}_{i=1}^M$ as $\underline{\mathbf{Y}}_k = \mathbf{I} \underline{\mathbf{e}}$, where $\underline{\zeta}_{k,i} = \langle \hat{\zeta}_{k,i}, \hat{\zeta}_{k,i} \rangle^{-1} \langle \underline{e}_j, \underline{e}_j \rangle \underline{e}_j + \underline{e}_i$ and $\underline{\mathbf{e}} = [\underline{e}_1^*, \dots, \underline{e}_M^*]^*$. The lower triangular matrix \mathbf{I} is $[\mathbf{I}]_{i,j} = \langle \hat{\zeta}_{k,i}, \hat{\zeta}_{k,i} \rangle^{-1} \langle \underline{e}_j, \underline{e}_j \rangle^{-1}$ for $j = 1 \dots i-1$ and $[\mathbf{I}]_{i,i} = 1$. The minimum value of the quadratic form (20) expressed in terms of the innovations is

$$J_M^{min}(\underline{\mathcal{T}}_0, z_k^M) = \underline{\mathbf{Y}}_k^* \mathbf{I}^{-1} \mathbf{R}_e^{-1} \mathbf{I}^{-*} \underline{\mathbf{Y}}_k = \underline{\mathbf{e}}^* \mathbf{R}_e^{-1} \underline{\mathbf{e}} \quad (22)$$

where $\mathbf{R}_e = \text{diag}(\mathbf{R}_{e,0} \dots \mathbf{R}_{e,M})$ and $\mathbf{R}_{e,i} = \langle \hat{\zeta}_{k,i}, \hat{\zeta}_{k,i} \rangle$. Thus \mathbf{R}_e can be estimated recursively from the Krein space projection of innovations. Recursive computation of \mathbf{R}_e leads to recursive computation of $J_M^{min}(\underline{\mathcal{T}}_0, z_k^M)$ as,

$$J_i^{min}(\underline{\mathcal{T}}_{0|i}, z_k^i) = J_{i-1}^{min}(\underline{\mathcal{T}}_{0|i-1}, z_k^{i-1}) + \underline{e}_i^* \mathbf{R}_{e,i}^{-1} \underline{e}_i \quad (23)$$

Therefore the condition for the existence of the H^∞ estimator is

$$J_M^{min}(\underline{\mathcal{T}}_0, z_k^M) = \sum_{i=1}^M \begin{bmatrix} e_{z,i} \\ e_{s,i} \end{bmatrix}^* \mathbf{R}_{e,i}^{-1} \begin{bmatrix} e_{z,i} \\ e_{s,i} \end{bmatrix} > 0 \quad (24)$$

where $\underline{e}_i = [e_{z,i}^*, e_{s,i}^*]^*$ and $e_{z,i} = (z_k[i] - \underline{H}_k[i] \underline{C}_k[i] - \hat{z}_k[i|i-1])$, $e_{s,i} = (\tilde{s}_{i|i} - \hat{s}_{i|i-1})$. State-space model (21) gives $\mathbf{R}_{e,i} = \langle \underline{\Theta}_k \tilde{c}_k, \underline{\Theta}_k \tilde{c}_k \rangle + \langle \bar{\eta}_k, \bar{\eta}_k \rangle$ where $\tilde{c}_k[i] = \bar{c}_k[i] - \hat{c}_k[i|i-1]$. Calculations similar to [7] leads to,

$$J_M^{min}(\underline{\mathcal{T}}_0, z_k^M) = \sum_{i=1}^M (z_k[i] - \underline{H}_k[i] \underline{C}_k[i] - \hat{z}_k[i|i-1])^* (1 + H_k P_k H_k^*)^{-1} (z_k[i] - \underline{H}_k[i] \underline{C}_k[i] - \hat{z}_k[i|i-1]) > 0 \quad (25)$$

which is a unique minimum and guarantees the existence condition (24) of a certain level- γ_f H^∞ filter. $\hat{z}_k[i|i-1] = H_k[i] \hat{c}_k[i|i-1]$. $\hat{c}_k[i|i-1] = F \hat{c}_k[i-1|i-1]$ and $\hat{c}_k[i-1|i-1]$ is calculated recursively using H^∞ filtering.

3.3. Robust Multiple Model Detector (RMMD)

The discrete-time Riccati solution for the H^∞ filter as given in [5], [6],

$$P_k[i+1] = P_k[i] - P_k[i] \begin{bmatrix} H_k^* & \theta_k^* \end{bmatrix} \mathbf{R}_{e,i}^{-1} \begin{bmatrix} H_k \\ \theta_k \end{bmatrix} P_k[i] + Q$$

$$\mathbf{R}_{e,i} = \mathbf{R} + \begin{bmatrix} H_k \\ \theta_k \end{bmatrix} P_k[i] \begin{bmatrix} H_k^* & \theta_k^* \end{bmatrix}; P_k[0] = P_0 \quad (26)$$

If $P_k[i]$ exists then one possible level- γ_f filter will be,

$$\begin{aligned} \tilde{s}_{:i} &= \theta_k[i] \hat{c}_k[i], \\ \hat{c}_k[i] &= \hat{c}_k[i] + K_k(z_k[i] - \underline{\mathcal{H}}_k[i] \underline{\mathcal{C}}_k[i] - H_k \hat{c}_k[i]) \\ \hat{c}_k[0] &= F \hat{c}_k[i-1] - 1; \hat{c}_k[0] = x_0, \\ K_k &= P_k[i] H_k^* (1 + H_k P_k[i] H_k^*)^{-1} \\ \underline{\mathcal{H}}_k[i] \underline{\mathcal{C}}_k[i] &= \underline{\mathbf{L}}_{1:k-1} \mathbf{A}_{1:k-1} \hat{\mathbf{B}}_{1:k-1} [i] \hat{\mathbf{c}}_{1:k-1} [i] \\ \hat{\mathbf{B}}_{1:k-1} [i] &= \text{diag}(\hat{b}_1[i], \dots, \hat{b}_{k-1}[i]) \\ \hat{\mathbf{c}}_{1:k-1} [i] &= [\hat{c}_1[i], \dots, \hat{c}_{k-1}[i]]^* \end{aligned} \quad (27)$$

The adaptive H^∞ filtering problem can have many solutions, each for a proper γ_f value. We adjust the γ_f value at each time i to satisfy the condition (24), that is,

$$\begin{aligned} -\gamma_f^2 + \theta_k(P_k^{-1} + H_k^* H_k)^{-1} \theta_k^* &< 0 \\ \gamma_f^2 &> \max\{\text{eig}[H_k(1 - K_k H_k) P_k H_k^*]\} \text{ for } \theta_k = H_k \\ \gamma_f &= \delta \max\{\text{eig}[H_k(1 - K_k H_k) P_k H_k^*]\}^{\frac{1}{2}} \end{aligned} \quad (28)$$

where $\delta > 1$ is a constant value large enough to ensure that γ_f is always greater than the maximum square root eigen value.

The recursive PI for a time sequence M is given by (25). At time instant i we may calculate $J_i^{min}(\cdot)$ recursively as,

$$J_i^{min}(\underline{\mathcal{I}}_{0:i}, z_k^i) = J_{i-1}^{min}(\underline{\mathcal{I}}_{0:i-1}, z_k^{i-1}) + \Delta J_{i|i-1} \quad (29)$$

where $\Delta J_{i|i-1} = (z_k[i] - \underline{\mathcal{H}}_k[i] \underline{\mathcal{C}}_k[i] - \hat{z}_k[i|i-1])^* (1 + H_k[i] P_k[i] H_k^*[i])^{-1} (z_k[i] - \underline{\mathcal{H}}_k[i] \underline{\mathcal{C}}_k[i] - \hat{z}_k[i|i-1])$. At each time instant, for each user the total number of models will be $\mathcal{M} = 2$, and there are 2 model-matched H^∞ filters operating in parallel each (denoted $q \in \mathcal{M}$) matched to a possible hypothesis $b_k^q \in \{-1, +1\}$. Each model calculates the associated PI, $\Delta J_{i|i-1}^q(\cdot)$, and the value of $\hat{b}_k[i]$ is given by the model q^{MIN} which is the model with the minimum PI. That is $\hat{b}_k[i] = b_k^{q^{MIN}}$ and the channel estimate is $\{\hat{c}_k^{q^{MIN}}[i], P_k^{q^{MIN}}[i]\}$. At next time instant, all model-matched filters are initialized with the channel estimates of the filter matched to q^{MIN} . Thus the prior estimates for equations (26)-(27) use the estimates from the model q^{MIN} at previous time-instant. Thus we can implement a symbol-by-symbol detector where decisions are made at each time instant based on $\Delta J_{i|i-1}^q(\underline{\mathcal{I}}_{0:i}, z_k^i, \hat{c}_k^{q^{MIN}}[i-1|i-1], P_k^{q^{MIN}}[i-1])$ for each user. The computational complexity is in the order of $O(MK[K + \mathcal{M}])$.

4. NUMERICAL COMPARISONS

We compare the performance of the proposed RMMD algorithm with the following two algorithms.

4.1. M-estimator based M-decorrelators (M-dec)

(6) can be written as,

$$\mathbf{r}[i] = \mathbf{S} \underline{\phi}[i] + \underline{\omega}[i] \quad (30)$$

where $\underline{\phi}[i] = [A_1 b_1[i] c_1^*[i] \cdots A_K b_K[i] c_K^*[i]]^*$. The basic idea is to detect the symbols in (8) by first estimating the complex vector $\underline{\phi}[i]$, and then extracting the symbol estimates from these continuous estimates as explained in [4]. The following synopsis lists the key steps involved. The estimates are obtained by using an estimator of the class of M -estimators (Maximum Likelihood Estimators) which minimize a function $\rho_H(\cdot)$ of the residuals,

$$\hat{\underline{\phi}}[i] = \arg \min_{\underline{\phi}[i] \in \mathcal{C}^K} \sum_{n=1}^N \{\rho_H[\Re(f)] + \rho_H[\Im(f)]\} \quad (31)$$

where $f = \mathbf{r}_n[i] - \sum_{l=1}^K [\underline{\mathcal{A}}]_{nl} \underline{\phi}_l[i]$ and the detected symbols are given by, $\hat{b}_k[i] = \text{sgn}\{\Re[\hat{\underline{\phi}}_k[i] \hat{\underline{\phi}}_k^*[i-1]]\}$. There are different choices for the penalty function but the choice of the authors in [4], [3] has been,

$$\rho_H(x) = \begin{cases} \frac{x^2}{2\sigma_n^2}, & \text{for } |x| \leq \xi \sigma_n^2 \\ -\frac{\xi^2 \sigma_n^2}{2} + \xi |x|, & \text{for } |x| > \xi \sigma_n^2 \end{cases}$$

where $\xi = 1.5/\sigma_n^2$. The computational complexity of M-dec is in the order of $O(MKN)$.

4.2. H^2 -based Multiuser Detector (IMM-SIC)

Similarly to the H^∞ -based method the multiple model based detector can be implemented using a H^2 -based approach and this paper employs $\mathcal{M} = 2$ Kalman filters operating in parallel. Each model-matched Kalman filter (denoted $q \in \mathcal{M}$) calculates the likelihood of each hypothetical model based on the predicted channel estimate $\{\hat{c}_{i|i-1}^q, P_{i|i-1}^q\}$ and input $z_k[i]$.

$$\begin{aligned} \hat{c}_{i|i-1}^q &= F \hat{c}_k^{0q}[i] \\ P_{i|i-1}^q &= F P_k^{0q}[i] F^* + Q \\ e_{z,i}^q &= z_k[i] - \underline{\mathcal{H}}_k[i] \underline{\mathcal{C}}_k[i] - H_k^q [i] \hat{c}_{i|i-1}^q \\ S_q &= H_k^q [i] P_{i|i-1}^q H_k^q [i]^* + \underline{\mathcal{H}}_k [i] \underline{\mathcal{P}}_k [i] \underline{\mathcal{H}}_k^* [i] + \rho^2 \\ \Delta_q [i] &= -\exp\{-e_{z,i}^q S_q^{-1} e_{z,i}^{q*}\} \end{aligned} \quad (32)$$

The model with the maximum likelihood is matched to the actual transmitted user symbol $b_k[i]$.

$$q^{MAP} = \arg \max_{q \in \mathcal{M}} \Delta_q [i]; \hat{b}_k [i] = b_k^{q^{MAP}} \quad (33)$$

The well known Interacting Multiple Model (IMM) algorithm is used where the mixing step gives the estimates $\{\hat{c}_k^{0q}[i], P_k^{0q}[i]\}$ from channel estimates $\{\hat{c}_k^q[i-1|i-1], P_k^q[i-1|i-1]\}$, generated from the parallel Kalman filters at the previous time-instant. Due to space restriction we refer the reader to [8] for further details on implementation of IMM and to [9] for joint channel estimation and data decoding using the IMM-SIC algorithm. The computational complexity of IMM-SIC is in the order of $O(MK[K + \mathcal{M}^2])$.

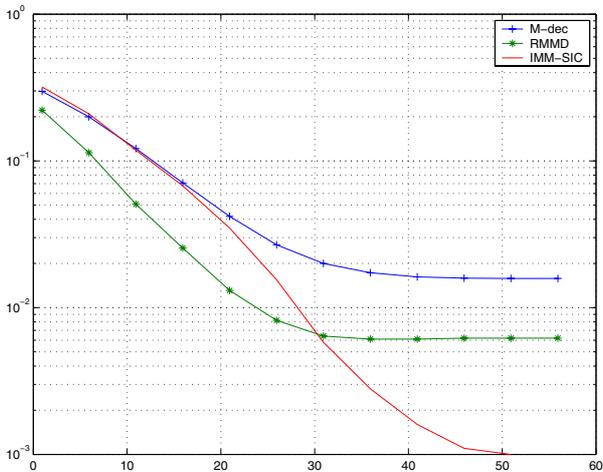


Fig. 1. BER vs. SNR for user 1, $f_D T = 0.0056$.

5. SIMULATION RESULTS

In simulations, a synchronous CDMA system with $K = 5$ users, in which each user is assigned an independent spreading sequence, is considered. The processing gain is $N = 31$. The fading channel is modeled as a lightly damped second-order autoregressive (AR) process

$$c_k[i] = -\beta_1 c_k[i-1] - \beta_2 c_k[i-2] + n_k[i] \quad (34)$$

where $c_k[i]$ is the $\{k\}$ -th channel fading coefficient in the i -th symbol interval, the driving noise process $n_k[i]$ is a zero-mean complex white Gaussian process, and the AR parameters are related to the physical parameters of the fading channel. Specifically, $\beta_1 = -2r_d \cos(2\pi f_p T)$, $f_p = f_d/\sqrt{2}$, $\beta_2 = r_d^2$, $r_d = 0.998$, is fixed where f_d is the Doppler Frequency and pole radius is r_d . In this case of channel fading, differential modulation must be used to overcome the resulting π radian phase ambiguity. Figures 1, 2 shows the BER performance for a BPSK transmission with differential decoding. Numerical experiments showed that on average M-dec took three iterations to obtain the best BER performance. $\delta = 10$, $\epsilon = 0.1$ and $\kappa = 100$. IMM-SIC and RMMD are *non-iterative* algorithms.

In the low SNR region of < 30 dB the RMMD algorithm outperforms the IMM-SIC algorithm. Both the RMMD and M-dec exhibits an error floor but the RMMD has better BER across the SNR range. The error floor seems to mainly depend on the fading correlation coefficient. Thus for high SNR, the detection performance appears to be relatively insensitive to choice of the noise distribution. Therefore for the RMMD and M-dec the slower the fading rate, the lower the error floor; the maximum-likelihood detector (IMM-SIC) is less sensitive to fading coefficient and as SNR increases the decoder becomes less sensitive to the noise distribution as well. Therefore the performance of IMM-SIC does not exhibit an error floor.

6. CONCLUSION

We have developed a new robust multiuser detection method for fading channels based on multiple models and H^∞ estimation.

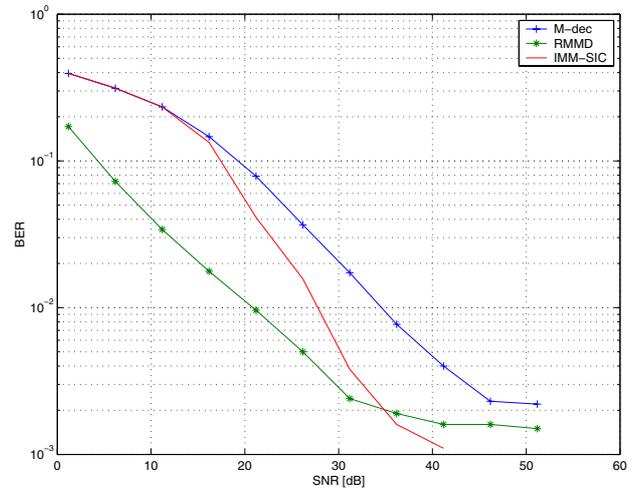


Fig. 2. BER vs. SNR, $f_D T = 0.00141$.

By using a worst-case PI and model switching, this method offers significant performance improvement over recently proposed iterative M-dec detector in impulsive noise.

7. REFERENCES

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