SPACE-TIME BLOCK CODING : JOINT DETECTION AND CHANNEL ESTIMATION USING MULTIPLE MODEL THEORY

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ABSTRACT

A joint decoding method for space-time block codes [1, 2] is presented. The space-time coded signals can be viewed as a first-order Markov chain, permitting the development of a Multiple Model algorithm. Therefore the quasi-static assumption [2] can be relaxed and joint channel and symbol estimation is carried out at each time-instant as opposed to at the end of a block duration.

1. INTRODUCTION

The wireless channel suffers attenuation due to multipath, interference of other users and channel fading, often statistically modeled as Rayleigh distributed, which makes detection of the transmitted signal at the receiver difficult. It was assumed that the channel is quasi-static flat fading and spatial diversity was achieved by deploying multiple antennae at the transmitter [2]. Early work which studied transmit diversity were [3, 4, 5]. More recently, space-time trellis coding was proposed by [2], which generalized the two transmitter scheme developed by [1], and provides significant gains over [3, 4]. These methods provide full spatial diversity and half or $3/4$ of the maximum possible transmission rate for specific signal constellations. They also have a very simple maximum likelihood decoding algorithm based only on linear processing at the receiver. Later work [6] considered performance results when channel state information was not known and same decoding criteria was used with two stage channel estimation and decoding. The limitation of the method in [6] is that it assumes the channel is quasi-static and that all received signals during one frame duration are independent. The bit-error-rate (BER) performance of the decoder when path gains vary rapidly is unclear.

The novelty of this paper is the development of a joint decoding strategy based on multiple model theory for space-time coded signals. Considering the Markovian properties of the transmitted signal a maximum a posteriori (MAP) symbol-by-symbol algorithm is proposed. The advantage of the space-time multiple model decoder (MMD) is that it can be implemented at each time instant as opposed to the maximum-likelihood decoder which decodes the transmitted symbols at the end of each block duration. Decoding at each time instant also means that higher transmission rates are possible but at the expense of decreased performance. By implementing a set of parallel filters, each matched to a system mode in the multiple model system a joint channel estimator and MAP symbol decoder can be developed. Extension of the work presented here to frequency-selective channels is straight forward.

2. SYSTEM MODEL

Consider a wireless system with N_T transmitters and N_R receivers. The symbol sequences $\{\beta_{n_s}[i]\}_{n_s=1}^{N_s}$, is sent to a space-time encoder and the encoded data goes through a serial-to-parallel converter and is divided into N_T streams. At each time instant i, the data stream $b_{n_t}[i]$ is transmitted from the antenna n_t. The signals $b_1[i], \cdots, b_{N_T}[i]$ are transmitted simultaneously and all symbols have a transmission period of symbol duration T. The signal received by antenna n_r at time i is given by,

$$\mathbf{r}^{n_r}[i] = \frac{1}{\sqrt{N}} \sum_{n_t=1}^{N_T} c^{n_t,n_r}[i]\mathbf{x}^{n_t}[i] + \mathbf{v}^{n_r}[i] \tag{1}$$

$$\begin{aligned} \text{where,} \quad \mathbf{x}^{n_t}[i] &= A^{n_t} b_{n_t}[i]\mathbf{s}^{n_s} \\ \mathbf{s}^{n_s} &= [s_1^{n_s}, \ldots s_N^{n_s}]^\top \\ \mathbf{v}^{n_r}[i] &= [v^{n_r}[iN]^*, \ldots, v^{n_r}[(i+1)N-1]^*]^* \\ \mathbf{r}^{n_r}[i] &= [r^{n_r}[iN]^*, \ldots, r^{n_r}[(i+1)N-1]^*]^* \end{aligned} \tag{2}$$

where N is the processing gain of the signature sequence $\mathbf{s}^{n_s}$ assigned to the n_sth information stream β_{n_s} and the real and imaginary components of the complex noise $v^{n_r}[iN+n]$ at time i are independent samples of a zero-mean Gaussian random variable with variance $\rho_v^2/2$. The complex path gains from transmit antenna n_t to receive antenna n_r, $c^{n_t,n_r}[i]$ includes the effects of the transmitter, receiver and the amplitude and phase of the channel response. The gains are modeled in Cartesian co-ordinate form in the complex plane ($c = c_R + jc_I$). The rapid fading remains constant over a symbol duration but changes at each time instant.

3. TRANSMISSION MODEL

The input symbol sequence $\{\beta_{n_s}[i]\}_{n_s=1}^{N_s}$ can be considered as generated from a Markov source. An i.i.d. source is a special case. Ψ_{n_s} is the transition probability matrix of $\beta_{n_s}[i]$ with dimension $S \times S$ where S is the size of the symbol alphabet of the data stream. When $\beta_{n_s}[i]$ is an i.i.d. information stream then each element of Ψ_{n_s} is equal to $P(\beta_{n_s}[i]|\beta_{n_s}[i-1]) = \frac{1}{S}$. Then the overall transition probability matrix for the vector $\beta[i]$ is given by $\Pi_0 = [\Psi_{N_s} \ldots \otimes \Psi_2 \otimes \Psi_1]_{\mathcal{Q}_s \times \mathcal{Q}_s}$ where $\mathcal{Q}_s = S^{N_s}$. $\otimes$ denotes Kronecker product operation. If the information source is not i.i.d. then Ψ_{n_s} is determined according to the Markov properties of the source. In the absence of any space-time coding the N_s information streams are transmitted simultaneously from $N_T = N_s$

transmit antennae and the transition probability matrix for the N_T transmitters is the same as the transition probability matrix for N_s information streams, i.e. Π_0. In this paper space-time encoding is considered at the transmitter end of the MIMO system and the next section explains the respective transmission models and the resulting transition probability matrices of encoded data streams.

3.1. Transition Matrices of Space-time Coded Signals

For any real constellation the STBC codes can achieve a maximum possible transmission rate of 1. For any complex constellation half this transmission rate can be achieved. The transmission rate of space-time codes is dependent on the block length p and the number of information streams N_s [2] and is given by $\frac{N_s}{p}$. In this paper, a multiple model symbol-rate decoder is derived which can decode the transmitted symbols at each time-instant instead of at the end of each block duration pT; leading to varying transmission rates from $\frac{N_s}{1}$, $\frac{N_s}{2}$ to $\frac{N_s}{p}$. Thus greater than one transmission rates can be achieved at the cost of reduced BER performance.

Each space-time block code is defined by a transmission matrix $\mathcal{G}_{N_T}$ with dimension $p \times N_T$ whose elements are linear combinations of variables $g_1, g_2, \ldots, g_{N_s}$ [2]. Code $\mathcal{G}_{N_T}$ uses p symbol durations to transmit N_s symbols, therefore the transmission rate R_{N_T} of this code will be $R_{N_T} = N_s/p$. The information stream $\{\beta_{n_s}\}_{n_s=1}^{N_s}$ is set to $\{g_{n_s}\}_{n_s=1}^{N_s}$ and transmitted according to $\mathcal{G}_{N_T}$ from N_T transmitters. The signal sequence transmitted from antenna n_t for the duration of a block length pT can be denoted as $b_{n_t}[ip], b_{n_t}[ip+1], \ldots, b_{n_t}[(i+1)p-1]$ and $\{g_{n_s}\}_{n_s=1}^{N_s}$ is set to $\{b_{n_t}[ip+j]\}_{n_t=1, j=0}^{N_T, \ p-1}$ according to the code $\mathcal{G}_{N_T}$.

For one transmit antenna the code is $\mathcal{G}_1 = [g_1]$, the information signal $\beta_1[i]$ is set equal to g_1 and transmitted from the single antenna. The transmitted symbol sequence is denoted as $b_1[i]$. This paper considers i.i.d. BPSK modulated anti-podal information sequence. Therefore the symbol alphabet of $\beta_1[i]$ is $\{-1, +1\}$ and $S = 2$. Now the size of the transmitted signal constellation is $\mathcal{Q} = S^{N_T} = 2$ and denoting a point in the signal constellation as $\bar{\mathbf{b}}_q \in \{-1, +1\}$ for $q = 1, 2$ then $b_1[i] \in \{\bar{\mathbf{b}}_1, \bar{\mathbf{b}}_2\}$. Therefore the elements of the transition probability matrix for the transmitted symbol stream $b_1[i] = \beta_1[i]$ is given in Table 1.

$\Pi_0(\mathcal{G}_1)$	$b_1[i-1]$	
$b_1[i]$	$\bar{\mathbf{b}}_1$	$\bar{\mathbf{b}}_2$
$\bar{\mathbf{b}}_1$	$\frac{1}{2}$	$\frac{1}{2}$
$\bar{\mathbf{b}}_2$	$\frac{1}{2}$	$\frac{1}{2}$

Table 1. Transition Probability Matrix $\Pi_0(\mathcal{G}_1) = \Psi_1$.

For two transmit antennae $N_T = 2$ the ST code $\mathcal{G}_2$ is [1],

$$\mathcal{G}_2 = \begin{pmatrix} g_1 & g_2 \\ -g_2^* & g_1^* \end{pmatrix} = \begin{pmatrix} \beta_1[i] & \beta_2[i] \\ -\beta_2^*[i] & \beta_1^*[i] \end{pmatrix} = \begin{pmatrix} b_1[2i] & b_2[2i] \\ b_1[2i+1] & b_2[2i+1] \end{pmatrix} \quad (3)$$

The first row of the transmission matrix is i.i.d. with respect to the previously transmitted information and its transition probability matrix is denoted as $\Pi_0(\mathcal{G}_2) = \Psi_2 \otimes \Psi_1$; the elements of which depend on the Markov properties of the original information source $\{\beta_{n_s}[i]\}_{n_s=1}^{N_s}$. The ST code introduces a first order Markov property with respect to the first row and the respective transition probability matrix will be denoted as $\Pi_1(\mathcal{G}_2)$. Hence for code $\mathcal{G}_2$, where $p = 2$, the transition probability matrices during the block duration pT at time instances $2i$ and $2i+1$, are $\Pi_0(\mathcal{G}_2)$ and $\Pi_1(\mathcal{G}_2)$ and are given in Tables 2 and 3 respectively. The size of the signal constellation is $\mathcal{Q} = S^{N_T} = 4$ and the signal constellation is $\bar{\mathbf{b}}_q \in \{-1-1, -1+1, +1-1, +1+1\}$ and $\mathbf{b}[i] = (b_1[i], b_2[i]) \in \{\bar{\mathbf{b}}_1, \ldots, \bar{\mathbf{b}}_\mathcal{Q}\}$. In this example, if

$\Pi_0(\mathcal{G}_2)$	$(b_1[2i-1], b_2[2i-1])$			
$(b_1[2i], b_2[2i])$	$\bar{\mathbf{b}}_1$	$\bar{\mathbf{b}}_2$	$\bar{\mathbf{b}}_3$	$\bar{\mathbf{b}}_4$
$\bar{\mathbf{b}}_1$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\bar{\mathbf{b}}_2$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\bar{\mathbf{b}}_3$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\bar{\mathbf{b}}_4$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Table 2. Transition Probability Matrix $\Pi_0(\mathcal{G}_2)$.

$\Pi_1(\mathcal{G}_2)$	$(b_1[2i], b_2[2i])$			
$(b_1[2i+1], b_2[2i+1])$	$\bar{\mathbf{b}}_1$	$\bar{\mathbf{b}}_2$	$\bar{\mathbf{b}}_3$	$\bar{\mathbf{b}}_4$
$\bar{\mathbf{b}}_1$	0	1	0	0
$\bar{\mathbf{b}}_2$	0	0	0	1
$\bar{\mathbf{b}}_3$	1	0	0	0
$\bar{\mathbf{b}}_4$	0	0	1	0

Table 3. Transition Probability Matrix $\Pi_1(\mathcal{G}_2)$.

$\bar{\mathbf{b}}_1 = [-1, -1]$ was transmitted from the two antennae at time $2i$ then the probability of transmitting $\bar{\mathbf{b}}_3 = [+1, -1]$ at time $2i+1$ is 1 and transmitting any other combination will be 0.

The first line of code $\mathcal{G}_2$ can be considered as composed of two instances of code $\mathcal{G}_1$. Therefore based on the fact that $\{g_{n_s}\}_{n_s=1}^{N_s}$ are orthogonal data streams [2], $\Pi_0(\mathcal{G}_2)$ can also be obtained by,

$$\Pi_0(\mathcal{G}_2) = \Pi_0(\mathcal{G}_1) \otimes \Pi_0(\mathcal{G}_1) \quad (4)$$

Next the examples of orthogonal designs for $N_T = 3$ and $N_T = 4$ transmit antennae is considered for $N_s = 4$ i.i.d. information streams. Considering the ST code $\mathcal{G}_3$,

$$\mathcal{G}_3 = \left(\begin{array}{cc|c} g_1 & g_2 & g_3 \\ -g_2 & g_1 & -g_4 \\ \hline -g_3 & g_4 & g_1 \\ -g_4 & -g_3 & g_2 \end{array} \right) \quad (5)$$

the transition probability matrix at time instant $4i$ when the first row of the code $\mathcal{G}_3$ in (5) is transmitted can be derived by,

$$\Pi_0(\mathcal{G}_3) = \Pi_0(\mathcal{G}_2) \otimes \Pi_0(\mathcal{G}_1) \quad (6)$$

due to the fact that this row is seen to be composed of one instance of the first row of code $\mathcal{G}_2$ and one instance of code $\mathcal{G}_1$. Similarly the other transition probability matrices at times $4i+1$ and $4i+3$ at which times the second and fourth rows in the code $\mathcal{G}_3$ are transmitted is given by,

$$\Pi_1(\mathcal{G}_3) = \Pi_3(\mathcal{G}_3) = \Pi_1(\mathcal{G}_2) \otimes \Pi_0(\mathcal{G}_1) \quad (7)$$

However the matrix formed by second and third rows and last two columns is composed of code $\mathcal{G}_2$ and the vector formed by the same rows and first column is composed of an instance of code

$\mathcal{G}_1$. Hence the transition probability matrix at time instance $4i+2$ is given by the reverse calculation,

$$\Pi_2(\mathcal{G}_3) = \Pi_0(\mathcal{G}_1) \otimes \Pi_1(\mathcal{G}_2) \quad (8)$$

For $N_T = 4$ the code for a real signal constellation is,

$$\mathcal{G}_4 = \left(\begin{array}{cc|cc} g_1 & g_2 & g_3 & g_4 \\ -g_2 & g_1 & -g_4 & g_3 \\ \hline -g_3 & g_4 & g_1 & -g_2 \\ -g_4 & -g_3 & g_2 & g_1 \end{array} \right) \quad (9)$$

The transition probability matrix at time instant $4i$ will be,

$$\Pi_0(\mathcal{G}_4) = \Pi_0(\mathcal{G}_2) \otimes \Pi_0(\mathcal{G}_2) \quad (10)$$

The transition probability matrix at time instant $4i+1$ will be,

$$\Pi_1(\mathcal{G}_4) = \Pi_1(\mathcal{G}_2) \otimes \Pi_1(\mathcal{G}_2) \quad (11)$$

The transition probability matrix at time instant $4i+2$ will be,

$$\Pi_2(\mathcal{G}_4) = \begin{bmatrix} \Pi_1(\mathcal{G}_2) \otimes \mathbf{W} & \Pi_1(\mathcal{G}_2) \otimes \mathbf{X} \\ \Pi_1(\mathcal{G}_2) \otimes \mathbf{Y} & \Pi_1(\mathcal{G}_2) \otimes \mathbf{Z} \end{bmatrix} \quad (12)$$

where $\Pi_1(\mathcal{G}_2) = \begin{bmatrix} \mathbf{W} & \mathbf{X} \\ \mathbf{Y} & \mathbf{Z} \end{bmatrix}$ is in patitioned form and $\mathbf{W}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}$ are square matrices of dimension 2×2. The transition probability matrix at time instant $4i+3$ will be,

$$\Pi_3(\mathcal{G}_4) = \Pi_1(\mathcal{G}_2) \otimes \Pi_1(\mathcal{G}_2) \quad (13)$$

Summarizing in the following table all the above transition probability matrices listed against their respective ST codes and rates R_{N_T} if decoding is done at each time instant as opposed to at the end of block length is,

$\mathcal{G}_{N_T}$	Transition Probability Matrices $\Pi_0(\cdot), \ldots, \Pi_{p-1}(\cdot)$	Code rates
$\mathcal{G}_1$	$\Pi_0(\mathcal{G}_1)$	1
$\mathcal{G}_2$	$\Pi_0(\mathcal{G}_2), \Pi_1(\mathcal{G}_2)$	$2, 1$
$\mathcal{G}_3$	$\Pi_0(\mathcal{G}_3), \Pi_1(\mathcal{G}_3), \Pi_2(\mathcal{G}_3), \Pi_3(\mathcal{G}_3)$	$4, 2, \frac{4}{3}, 1$
$\mathcal{G}_4$	$\Pi_0(\mathcal{G}_4), \Pi_1(\mathcal{G}_4), \Pi_2(\mathcal{G}_4), \Pi_3(\mathcal{G}_4)$	$4, 2, \frac{4}{3}, 1$

Extension of this formulation to $N_T > 4$ is straightforward. Generalizing to any number of transmitters the transition probability matrix at time instant pi can be written as,

$$\Pi_0(\mathcal{G}_{N_T}) = \begin{cases} \Pi_0(\mathcal{G}_{N_T/2}) \otimes \Pi_0(\mathcal{G}_{N_T/2}) & \text{for even } N_T. \\ \Pi_0(\mathcal{G}_{(N_T+1)/2}) \otimes \Pi_0(\mathcal{G}_{(N_T-1)/2}) & \text{for odd } N_T. \\ \frac{1}{\mathcal{Q}}\mathbf{1}_{\mathcal{Q}} & \text{i.i.d. sequence.} \end{cases}$$

where $\mathbf{1}_{\mathcal{Q}}$ is a square matrix of 1s with dimension $\mathcal{Q}$. The matrices $\Pi_1(\mathcal{G}_{N_T}), \Pi_2(\mathcal{G}_{N_T}), \ldots$, are derived based on the Markov property induced in the transmit sequence by the code $\mathcal{G}_{N_T}$.

4. THE RECEIVER MODEL

The receivers are spaced sufficiently apart to ensure that there is no correlation between received signals. Therefore the received signals given in (1) are stacked to obtain the receiver model below,

$$\mathbf{r}[i] = \sum_{n_t=1}^{N_T} \mathbf{S}^{n_t}(A^{n_t} b_{n_t}[i]\mathbf{I}_{N_R})\mathbf{c}^{n_t}[i] + \mathbf{v}[i] \quad (14)$$

Written in matrix form,

$$\mathbf{r}[i] = \mathbf{SAB}[i]\mathbf{c}[i] + \mathbf{v}[i] \quad (15)$$

where $\mathbf{B}[i] = \text{diag}(b_1[i], \ldots, b_{N_T}[i]) \otimes \mathbf{I}_{N_R}$, $\mathbf{A} = \text{diag}(A^1, \ldots, A^{N_T}) \otimes \mathbf{I}_{N_R}$, $\mathbf{S}^{n_s} = \frac{1}{\sqrt{N}}(\mathbf{I}_{N_R} \otimes \mathbf{s}^{n_s})$ and $\mathbf{c}[i] = [\mathbf{c}^1[i]^H \ldots \mathbf{c}^{N_T}[i]^H]^H$. The matched filter output is given below,

$$\mathbf{y}[i] = \mathbf{S}^T\mathbf{r}[i] = \mathbf{RAB}[i]\mathbf{c}[i] + \tilde{\mathbf{v}}[i] \quad (16)$$

where $\mathbf{y}[i] = [y^{(1,1)}, \ldots, y^{(1,N_R)}, y^{(N_T,1)}, \ldots, y^{(N_T,N_R)}]^H$. Therefore the $[(n_t-1)N_R + n_r]$ th component is given by,

$$y^{(n_t,n_r)}[i] = \Big[\underline{0}_{(n_r-1)}\, R_{n_r}^{(n_t,1)}\, \underline{0}_{N_R-1}\, R_{n_r}^{(n_t,2)} \ldots \underline{0}_{N_R-1} \\ R_{n_r}^{(n_t,N_T)}\, \underline{0}_{N_R-n_r}\Big] \mathbf{AB}[i]\mathbf{c}[i] + \tilde{v}^{(n_t,n_r)}[i]$$

where $R_{n_r}^{(n_t,n_t')}$ is the correlation between (n_t, n_t') antennae at the n_rth receiver. The output signal is input to the noise whitening filter $\mathbf{U}^{-1}$, obtained by upper-lower Cholesky factorization of $\mathbf{R} = \mathbf{UL}$, giving the matched filtered noise whitened output,

$$\mathbf{z}[i] = \mathbf{U}^{-1}\mathbf{y}[i] = \mathbf{LAB}[i]\mathbf{c}[i] + \bar{\mathbf{v}}[i] \quad (17)$$

where $\mathbf{z}[i] = [z^{(1,1)}, \ldots, z^{(1,N_R)}, z^{(N_T,1)}, \ldots, z^{(N_T,N_R)}]^H$. Therefore the $[(n_t-1)N_R + n_r]$ th component is given by,

$$z^{(n_t,n_r)}[i] = \Big[\underline{0}_{(n_r-1)}\, L_{n_r}^{(n_t,1)}\, \underline{0}_{N_R-1}\, L_{n_r}^{(n_t,2)} \ldots \underline{0}_{N_R-1} \\ L_{n_r}^{(n_t,n_t)}\, \underline{0}_{(N_T-n_t+1)N_R-n_r}\Big] \mathbf{AB}[i]\mathbf{c}[i] + \bar{v}^{(n_t,n_r)}[i] \quad (18)$$

where $\bar{v}^{(n_t,n_r)}[i]$ is white noise with variance ρ_v^2.

5. HYBRID SYSTEM APPROACH TO THE STC MODEL

The objective is to find the transmitted symbol sequence from measurement sequence (17). The rapidly time-varying channel is tracked by,

$$\mathbf{c}[i+1] = \mathbf{Fc}[i] + \mathbf{w}[i] \quad (19)$$

for some known $\mathbf{F}$,(typically with $\mathbf{F} = f\mathbf{I}_{N_R N_T}$ for some scalar $0 \ll f < 1$ [8]). Note that $f = 1$ will result in a Random-Walk model. $\mathbf{w}(i)$ is driving disturbance which is statistically independent and Gaussian distributed i.e., $\mathcal{N}(0, \rho_w^2\mathbf{I}_{N_R N_T})$. The MIMO system state-space model, (19),(17), is posed as a hybrid system and through state estimation the system's behavior is tracked along both its continuous state (channel gain) changes and its discrete state (symbol) changes. It is common to refer to the discrete state of the hybrid system as system's mode.

5.1. Hybrid system representation

Considering a (N_T, N_R) MIMO system with $p \times N_T$ space-time block code the size of the signal constellation is $\mathcal{Q} = S^{N_T}$. Therefore the vector $\mathbf{b}[i]$ can take one of $\mathcal{Q}$ possible values.

$$\mathbf{b}[i] = [b_1[i], \ldots, b_{N_T}[i]] \in \{[-1,-1,\ldots,-1], \ldots \\ ,[+1,+1,\ldots,+1]\} \equiv \{\bar{\mathbf{b}}_1, \ldots, \bar{\mathbf{b}}_{\mathcal{Q}}\} \quad (20)$$

where $\bar{\mathbf{b}}_q$ represent a possible transmitted symbol vector and is a possible mode of the system. This system has N_T independent transmission streams at each time-instant therefore the number of modes at each time instant is the same as the size of the signal constellation. If the mode index is given by q then $q \in 1, \ldots, \mathcal{Q}$ and the mode at time instant i, $m[i]$ is equal to m_q; written in short as m_q^i. Mode m_q^i represents the transmitted symbol vector $\bar{\mathbf{b}}_q$. The measurement (17) differs for each system mode and the

Table 4. IMM Algorithm for MIMO STC System.

IMM is initialized at i=0 with $P(m_q^0|\mathbf{Z}^0) = \frac{1}{\mathcal{Q}}$ and $\mathbf{c}_q[0] \sim \mathcal{N}(\bar{\mathbf{c}}_0, \bar{\mathbf{\Sigma}}_0)$;
For sequence length $i = 1, \ldots, M$ the IMM is called recursively.
FOR $t = pi : p(i+1) - 1$ DO

All Kalman filters are initialized with mixed channel estimates.
Mixing probabilities (weights) $\mu_{q'|q}[t]$ is used in calculating the mixed estimates (or prior distributions),
$p(\mathbf{c}[t]|m_q^t, \mathbf{Z}^{t-1}) \sim \mathcal{N}(\hat{\mathbf{c}}_q^0[t], \mathbf{\Sigma}_q^0[t])$ for $\forall q$.
$P(m_{q'}^{t-1}|m_q^t) = \pi_{qq'}$ where $\pi_{qq'} = [\Pi_j]_{qq'}$ for $t = pi + j$ and $j = 0, \ldots, p-1$ respectively.
FOR $q = 1, \ldots, \mathcal{Q}$ DO

Compute one-step predictive update for qth Kalman filter,
$\hat{\mathbf{c}}_q[t|t-1] = \mathbf{F}\hat{\mathbf{c}}_q^0[t]$;
$\mathbf{\Sigma}_q[t|t-1] = \mathbf{H}_q[t]\mathbf{\Sigma}_q^0[t]\mathbf{H}_q[t]^\top + \rho_w^2\mathbf{I}_{N_T N_R}$
Compute innovation likelihood,
$p(\mathbf{z}[t]|m_q^t, \mathbf{Z}^t) \sim \mathcal{N}(\epsilon_q, \mathbf{S}_q)$ with $\epsilon_q, \mathbf{S}_q$
Compute one-step Kalman filtering update,
$\hat{\mathbf{c}}_q[t|t] = \mathbf{E}\{\mathbf{c}_q[t]|m_q^t, \mathbf{Z}^t\}$;
$\mathbf{\Sigma}_q[t|t] = \text{Cov}\{\mathbf{c}_q[t]|m_q^t, \mathbf{Z}^t\}$.

END
Compute all the mode probabilities;
$\mu_q[t] = P(m_q^t|\mathbf{Z}^t)$.
MAP estimation; $q^{MAP} = \arg\max_q \mu_q[t]$ and $\hat{\mathbf{b}}[t] = \bar{\mathbf{b}}_{q^{MAP}}$
IF mod(t,p) == 1,

Symbol detection :
$\left[\begin{array}{ccc} \hat{\beta}_1[i] & \ldots & \hat{\beta}_{N_s}[i] \end{array}\right]^\top$
$= \mathbf{P}_{j+1}\left[\begin{array}{ccc} \hat{b}_1[p(i+1)-1] & \ldots & \hat{b}_{N_s}[p(i+1)-1] \end{array}\right]^\top$

END

END

q-th hypothesis at time $(pi+j)$ where $j = 0, \ldots, p-1$, is given by,

$$h_q: \quad \mathbf{z}[pi+j] = \mathbf{L}[pi+j]\mathbf{A}\mathbf{B}_q\mathbf{c}[pi+j] + \bar{\mathbf{v}}[pi+j] \quad \text{if system mode at time-instant } (pi+j) \text{ is } m_q^{pi+j}. \tag{21}$$

where the transmitted symbol matrix at time instant $(pi+j)$, $\mathbf{B}[pi+j] = \mathbf{B}_q = \text{diag}(\bar{\mathbf{b}}_q) \otimes \mathbf{I}_{N_R}$ and $\mathbf{B}_q$ is given by the 'symbol-mode' association $\{\bar{\mathbf{b}}_q, m_q^{pi+j}\}$ for $q = 1, \ldots, \mathcal{Q}$. The channel model is,

$$\mathbf{c}[pi+j] = \mathbf{F}\mathbf{c}[pi+j-1] + \mathbf{w}[pi+j-1] \tag{22}$$

Hence (21),(22) gives the hybrid system representation pertaining to each possible mode m_q^{pi+j} (hypothesis h_q) that a system could be in at a given time. A more detailed description of hybrid system approach for communication can be found in [9]. The transition probability matrices for the block duration of length pT will be,

$$\left.\begin{array}{rcl} P(m_q^{pi}|m_{q'}^{pi-1}) &=& [\Pi_0]_{qq'} \\ P(m_q^{pi+1}|m_{q'}^{pi}) &=& [\Pi_1]_{qq'} \\ &\vdots& \\ P(m_q^{(i+1)p-1}|m_{q'}^{(i+1)p-2}) &=& [\Pi_{p-1}]_{qq'} \end{array}\right\} \tag{23}$$

6. MULTIPLE MODEL BASED JOINT DECODER

Space-time codes [2] were designed for quasi-static flat Rayleigh or Rician fading, where CSI is available at the receiver. In practice perfect CSI is unavailable and [6] derived a maximum-likelihood decoding rule with a Viterbi decoder under the assumption that elements of $\mathbf{z}[i] = [\mathbf{z}[pi], \ldots, \mathbf{z}[p(i+1)-1]]$ are independent during one block duration. This assumption is inaccurate [7] and the proof is approximate and holds only in asymptotically high SNR. The IMM approach considers the Markovian property between the elements of $\mathbf{z}[i]$ induced by the STBC. A multiple model framework with number of system modes determined by the transmission model of the communication system was presented. The mode conditioned prior distribution of the channel state is approximated such that the past is summarized by a mode conditioned weighted channel state estimate and covariance. The weights at time instant $pi+j$ are the predicted mode probabilities $P(m_{q'}^{pi+j-1}| m_q^{pi+j}, Z^{pi+j-1})$ given posterior mode probabilities $P(m_{q'}^{pi+j-1}| Z^{pi+j-1})$ from the previous time instant. The mode conditioned posterior channel density function is,

$$p(\mathbf{c}[pi+j]|m_q^{pi+j}, \mathbf{Z}^{pi+j}) = \frac{p(\mathbf{z}[pi+j]|m_q^{pi+j}, \mathbf{c}[pi+j])}{p(\mathbf{z}[pi+j]|m_q^{pi+j}, \mathbf{Z}^{pi+j-1})} p(\mathbf{c}[pi+j]|m_q^{pi+j}, \mathbf{Z}^{pi+j-1}) \tag{24}$$

conditioned on modes at time-instant $pi+j$. The denominator is the innovation likelihood of the filter. Prior $p(\mathbf{c}[pi+j]|m_q^{pi+j}, \mathbf{Z}^{pi+j-1})$ in (24) follows from the Chapman-Kolmogorov equation as, $p(\mathbf{c}[pi+j]|m_q^{pi+j}, \mathbf{Z}^{pi+j-1}) = \int p(\mathbf{c}[pi+j]|\mathbf{c}[pi+j-1], m_q^{pi+j})p(\mathbf{c}[pi+j-1]|m_q^{pi+j}, \mathbf{Z}^{pi+j-1})d\mathbf{c}[pi+j-1]$. Channel variation is independent of transmitted symbols, $p(\mathbf{c}[pi+j]|\mathbf{c}[pi+j-1], m_q^{pi+j}) = p(\mathbf{c}[pi+j]|\mathbf{c}[pi+j-1])$. The Gaussian assumption leads to $p(\mathbf{c}[pi+j]|\mathbf{c}[pi+j-1]) \sim \mathcal{N}(\mathbf{c}[pi+j];\ \mathbf{F}\mathbf{c}[pi+j-1],\ \rho_w^2\mathbf{I}_{N_R N_T})$. Hence the channel estimate can be calculated using a parallel set of mode-matched Kalman filters. The previous channel estimate is substituted for by the mixed estimate of the mode conditioned posterior estimates from the previous time. This is determined as,

$$\begin{aligned} p(\mathbf{c}[pi+j-1]|m_q^{pi+j}, \mathbf{Z}^{pi+j-1}) &= \sum_{q'=1}^{\mathcal{Q}} p(\mathbf{c}[pi+j-1]| m_q^{pi+j}, m_{q'}^{pi+j-1}, \mathbf{Z}^{pi+j-1}) \underbrace{P(m_{q'}^{pi+j-1}|m_q^{pi+j}, \mathbf{Z}^{pi+j-1})}_{\mu_{q|q'}[pi+j]} \\ &\approx \sum_{q'=1}^{\mathcal{Q}} p(\mathbf{c}[pi+j-1]|m_q^{pi+j}, m_{q'}^{pi+j-1}, \hat{\mathbf{c}}_{q'}[pi+j-1|pi+j-1], \Sigma_{q'}[\cdot])\mu_{q|q'}[pi+j] \end{aligned} \tag{25}$$

The mixing probabilities are calculated as,

$$\mu_{q|q'}[pi+j] = \frac{1}{a} \underbrace{P(m_q^{pi+j}|m_{q'}^{pi+j-1})}_{[\Pi_j]_{qq'}} P(m_{q'}^{pi+j-1}|Z^{pi+j-1}) \tag{26}$$

and the mode probability update calculation is,

$$P(m_q^{pi+j}|Z^{pi+j}) = \frac{1}{a}p(\mathbf{z}[pi+j]|m_q^{pi+j}, Z^{pi+j}) \sum_{q'} \Pi_j(\mathcal{G}_{N_T})]_{qq'} P(m_{q'}^{pi+j-1}|Z^{pi+j-1})$$

The data decoding step is given by,

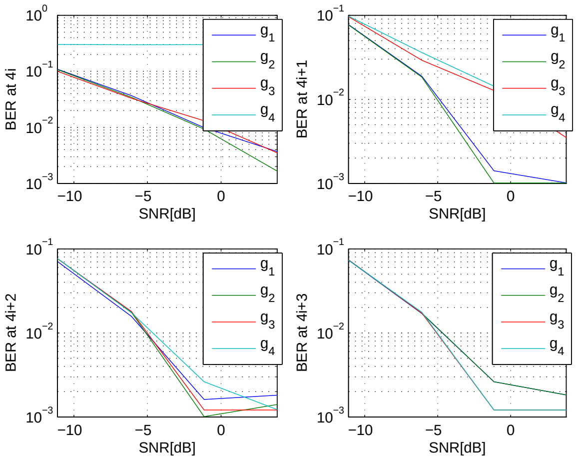

Fig. 1. BER vs. SNR for $\mathcal{G}_3$ and $N_R = 2$.

$$q^{MAP} = \arg\max_{q} P(m_q^{pi+j}|Z^{pi+j}) \quad (27)$$

$$\hat{\mathbf{b}}[pi+j] = \bar{\mathbf{b}}_{q^{MAP}} \quad (28)$$

$$\begin{bmatrix} \hat{\beta}_1[i] \\ \vdots \\ \hat{\beta}_{N_s}[i] \end{bmatrix} = \mathbf{P}_{j+1} \begin{bmatrix} \hat{b}_1[pi+j] \\ \vdots \\ \hat{b}_{N_s}[pi+j] \end{bmatrix} \quad (29)$$

When $N_T < N_s$ as in code $\mathcal{G}_3$, the decoder assumes that in addition to the N_T transmitters further $(N_s - N_T)$ 'virtual' transmitters so that at the decoder the multiple model framework is defined for N_s transmitters and code $\mathcal{G}_{N_s}$ is assumed. Therefore $\mathbf{P}_{j+1}$ is determined by the space-time code $\mathcal{G}_{N_T}$ $(N_s = N_T)$ or $\mathcal{G}_{N_s}$ $(N_s > N_T)$. The algorithmic details are given in Table 4.

7. SIMULATION RESULTS

The performance of the joint MMD is analyzed through Monte-Carlo simulations. A single-user Rayleigh flat-fading CDMA channel with processing gain $N = 31$ is considered. Each independent information source β_{n_s}, is assigned an independent signature waveform. BPSK modulation is employed and the i.i.d. BPSK data stream is obtained by generating binary random i.i.d. signals with uniform distribution. The complex channel gains are generated by passing two zero-mean Gaussian signals through a third-order Butterworth filter with fading rate $f_D T = 0.03$ to obtain rapidly time-varying complex channel gains. The driving noise variance was a design criterion and was chosen as $\rho_w^2 = 2 \times 10^{-3}$. Pilot symbols are used only to counteract the phase ambiguity present in joint channel estimation and symbol detection algorithms and not to obtain estimates of the fading channel gain. The number of pilot symbols used were $N_p = 7$ to the information symbols $N_i = 20$. Use of pilot symbols can be avoided by using differential encoding/decoding. The N_s independent symbols $\beta_1[i], \ldots, \beta_{N_s}[i]$ are decoded simultaneously at each time instant $pi + j$ with improved BER as j increases from 0 to $p - 1$. Having additional receivers offers data redundancy thus improving the performance results. Results for space-time code $\mathcal{G}_3$ and $N_R = 2$ is obtained here. Figures 1 and 2 shows the BER performance results of the joint MMD. The average BER over all the data streams

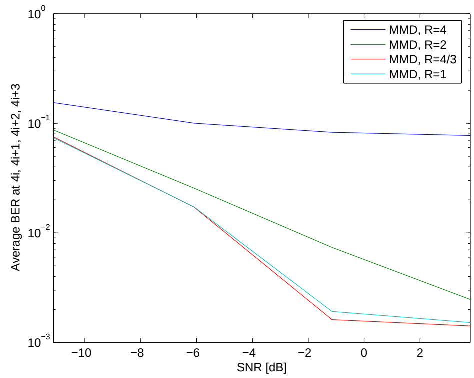

Fig. 2. Average BER vs. SNR for $\mathcal{G}_3$ and $N_R = 2$.

improves over the block duration from time $4i$ to $4i + 3$.

8. CONCLUSIONS

Thus this method can be used to decode the transmitted symbols at each time instant (giving an increased transmission rate, maximum of $R = \frac{N_s}{1}$) but with reduced performance. Most importantly this method enables joint channel estimation and symbol decoding of STC signals in rapidly time-varying channels.

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